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On the evaluation of the Eshelby tensor for polyhedral inclusions of arbitrary shape

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Abstract

We derive the analytical expression of the Eshelby tensor field for inclusions of arbitrary polyhedral shape. The formula contributed in the paper is directly expressed as function of the coordinates defining the vertices of the polyhedron thus avoiding the use of complex variables and anomalies exploited in previous contributions on the subject. It has been obtained by evaluating analytically the integrals appearing in the very definition of the Eshelby tensor by means of two consecutive applications of the Gauss theorem.

The first one allows one to express the original volume integrals as a sum of 2D integrals extended to the faces of the polyhedron, while the second application transforms each 2D integral into the line integrals extended so the edges of each face.

The effectiveness of the proposed formulation is numerically assessed by comparing the results provided by its implementation in a Matlab code with results available in the literature.

Keywords: Eshelby tensor, Polyhedral inclusion, Micromechanics, Analytical solution.

1. Introduction

The presence of inclusions in engineering materials affects their elastic fields, at the local and global scale, thus greatly influencing their mechanical and physical properties.

In particular composite materials take advantage of inclusions as reinforcements in the matrix in order to achieve superior properties that, otherwise, could not be obtained by individual constituent materials [1].

In other cases inclusions are unintentionally but inevitably formed during the material manufacturing process, such as oxides, carbides and voids in steel, and act as sources of stress concentration that affect performance and endurance of the material. Hence the study of inclusions plays an important role in the development of advanced materials for aerospace, marine, automotive and several additional applications.

Inclusions are usually categorized [2, 3] into homogeneous, inhomogeneous and inhomogeneities. Homogeneous inclusions have the same elastic moduli as the surrounding host or matrix material but contain the so called eigenstrain, i.e. a non elastic strain such as thermal expansion [4, 5] , phase transformation [6], visco-plastic strain [7] misfit strain [8] of quantum dot structures [9].

An inhomogeneous inclusion not only contains an eigenstrain but has also elastic moduli different from those of the matrix. Differently from the previous case an inhomogeneity does not contain an eigensrain.

The study of inclusions was pioneered by Eshelby in a celebrated paper [10] in which he considered an ellipsoidal inhomogeneous inclusion in an infinite matrix by simulating it as a homogeneous inclusion with an initial eigenstrain plus a properly selected equivalent eigenstrain. The ellipsoidal inclusion had uniform eigenstrain and stress if the initial eigenstrain within it was uniform.

The basic tool of this approach, denominated Equivalent Inclusion Method (EIM), was the Eshelby tensor, i.e. a fourth-order tensor relating the total strain to the eigenstrain, expressed as second derivative of the Green function of the elastic medium.

The EIM has allowed the effects of inclusions to be studied extensively and has been summarized in classical references, such as the review papers by Mura and co-workers [11, 12, 13, 14] and in several books [15, 16, 17, 18, 19]. Comprehensive reviews addressing applications of the EIM in micromechanics and homogeneization can be found in [20, 21, 22, 23].

Of particular relevance is the combination of the Eshelby theory with the Mori-Tanaka approach [24] for an accurate prediction of the effective thermomechanical properties of composites. Actually, just to quote some of the most recent contributions on the subject, the Eshelby-Mori-Tanaka mean-field theory has been applied in several contexts ranging from damage mechanics of cement concrete [25], to periodic composites [26], to micromechanical models of carbon nanotube composites [27, 28, 29, 30] to multi-coating micromechanics accounting for interphase behaviour [31, 32], to the analysis of thermal stresses developed in ceramic matrix composites [33] or hybrid nanocomposites [34], to studies concerning the influence of size effects in the evaluation of macroscopic properties of multifunctional nanocomposites [35, 36].

Although Eshelby tensor theory has been conceived for elliptical (ellipsoidal) inclusions, mainly to simplify the analytical treatment, non-elliptical (non-ellipsoidal) inclusions are of great practical importance for applications [37]. For instance polygonal or polyhedral SiC whiskers are used in metal and ceramic matrix composites, or eutectics in superconductor composites. Furthermore several modern semiconductor devices, such as lasers, infrared detectors and information storage de-