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Effective shear modulus of solids reinforced by randomly oriented- / aligned-elliptic multi-coated nanofibers in micropolar elasticity

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ABSTRACT

Accurate estimation of the in-plane shear modulus of solids reinforced by nano-/micro-size elliptical multi-coated fibers is the focus of this paper. It is well-known that at the scales comparable to the nanoscopic length scales of the material, traditional theory of elasticity ceases to hold and, moreover, due to lack of consideration of such length scales has an innate weakness of sensing the size effect. Therefore, it is proposed to formulate and calculate the effective shear modulus of the nano-/micro-composite within micropolar theory which introduces two material characteristic lengths into the field equations. For this purpose, Mori-Tanaka theory is extended to treat nested multi-inhomogeneity system in the mathematical framework of micropolar elasticity. The effective shear modulus pertinent to two cases of composites with aligned and randomly distributed enrichments is addressed. All the constituent phases are assumed to be micropolar media. As it will be seen, the estimations via couple stress and classical theories serve as the bounds of the estimate obtained using micropolar theory. The effects of the size and volume fraction of the fiber ensemble, the characteristic lengths and rigidity of the constituent phases, thickness of the coating layer, and the aspect ratio of the fiber ensemble on the effective shear modulus of the composite are examined.

1. Introduction

Fabrication of multifunctional nanocomposites having certain mechanical, optical, electrical, and magnetic features are among the major concerns in the industry - because of the demand for further developments and subsequent improvements of the ever growing modern technologies. Normally, by addition of about 0.5% to 5% by weight of nano-fiber or nano-particle of certain properties to a matrix one can obtain a nanocomposite with features of interest. Furthermore, the effective properties of such nanocompsites can be significantly altered by the employment of the coating technology to the additives. Recently, carbon nanotube and graphene have been used as enrichments to achieve composites with enhanced mechanical properties. Accordingly, some related experiments have been performed in the literature. For example, Rousakis et al. [1] have studied the effects of carbon nanotubes enrichment of epoxy resins on hybrid FRP-FR confinement of concrete. Liang et al. [2] addressed tensile properties of graphene nanoplatelets reinforced polypropylene composites. Meng and Khayat [3] studied the mechanical properties of ultra-high-performance concrete enhanced with graphite nanoplatelets and carbon nanofibers. Feng et al. [4] considered the nonlinear bending of polymer nanocomposite beams reinforced with non-uniformly distributed graphene platelets. Nano-sized additives, due to their large surface-to-volume ratio bond to their surrounding matrix differently than the usual enrichments which are about two order of magnitude larger. A critical review on the interfacial bonding strength encountered in nanotube-/nanoclay-polymer composites is given by Lau et al. [5]. The fiber-/particle-matrix interfacial conditions and properties can remarkably influence the overall behavior of the composite. In fact, the effective strength and toughness of nano-composite system can be significantly improved through the implication of the coating technology to the constituent nano-sized enrichments. It is of particular interest to develop suitable analytical methods for accurate prediction of the overall behavior of composites enriched with elliptical multi-coated nano-sized fibers. At such scales, classical theories due to lack of incorporation of the materials internal length scales cease to hold.

One of the innate weaknesses of traditional theory of elasticity is due to its lack of accuracy in providing the elastic fields of a nano-sized inhomogeneity. The size independency of the solutions obtained using methods which are based on traditional theory of elasticity is another

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shortcoming of this theory. To circumvent such dilemmas various higher order continuum theories have been developed in the literature. Formulations within such theories involve one or more material characteristic lengths which are linked to the discrete nature of matters. In the literature, few efforts have been given towards the calculation of material characteristic lengths of some crystalline solids within different augmented continuum theories [6–10]. It is due to the appearance of these internal length scales in the formulations that makes augmented continuum theories suitable not only for capturing certain size dependent effects, but also for sharpening the accuracy of the solutions. In the literature, there are an abundant amount of contributions which have focused on the prediction of the effective moduli of composites using different approaches, but based on the size independent traditional theory of elasticity; see, for example, [11–16]. For a fairly meticulous scrutiny of the literature on the subject, one may refer to [17–19].

Barretta et al. [20] examined Timoshenko nannobeams in the framework of Eringen-like constitutive law, in which they introduced two material length scales into their formulations. Later, Barretta et al. [21] modified nonlocal theory given by Eringen and Edelen [22] through incorporation of the first gradient of the axial and shear strain into their formulations. Then they studied the behavior of a functionally graded Timoshenko nanobeam within the modified theory. Pahlevani and Shodja [23] considered the surface and interface effects in the analysis of Saint-Venant torsion problem of an eccentrically two-phase fcc circular nanorod in the context of surface/interface elasticity. Barretta et al. [24] investigated elastic equilibrium of two-phase random composite beams under torsion, with simply and multiply connected cross-sections. Apuzzo et al. [25] provided an enhanced model of nonlocal torsion for nanobeams in the framework of Eringen theory. Furthermore, they derived the governing differential equations and the corresponding boundary conditions associated to torsion of nanobeams, and subsequently studied the size-dependent static torsional behavior of the proposed model. Along this school of thought, the examination of the phenomenon of size effect relevant to nano-beams and carbon nanotubes has been the focus of several studies [26–29]. Boutin [30], by using the homogenization method, provided an approach for the studying of the static microstructural effects of periodic elastic composites. This theory was later improved by Luciano and Willis [31,32]. Lusiano and Willis [31] argued that the mass density at a field point depends on the material at that point. They mentioned that, introduction of such a field is a necessity when dealing with porous media. Luciano and Willis [33] considered a solid reinforced by randomly distributed fibers and introduced a combination of the deterministic and configuration-dependent body forces. They accounted for the mean response of the material by introducing several non-local effective constitutive operators. Bisegna and Luciano [34] have provided bounds on the overall properties of composites with debounded frictionless interfaces. Haftbaradaran and Shodja [35] studied elliptic inhomogeneities and inclusions subjected to anti-plane loading within couple stress theory. They modified Mori-Tnaka's method [36] in the context of this theory and, moreover, under this type of loading they calculated the effective anti-plane shear moduli of solids reinforced with aligned single-phase elliptic nano-fibers. They captured the dependency of the effective anti-plane shear moduli on the size of the cross–sectional area of the fibers. Recently, Shodja and Alemi [37], for the in-plane case reformulated Mori-Tanaka's method in the mathematical framework of couple stress theory and predicted the effective in-plane shear modulus of solids reinforced by aligned as well as randomly oriented elliptic nanofibers.

The present study considers micropolar solids reinforced by a distribution of either aligned or randomly oriented micropolar elliptic multi-coated fibers. Accordingly, Mori-Tanaka's method is extended to estimate the effective in-plane shear modulus of n-phase composite within micropolar theory. To this end, at first the fundamental formulation of micropolar theory is briefly reviewed in Section 2. Afterwards, the governing equations pertinent to plane strain problems in elliptic coordinates (ξ, η) are presented. The class of a general solution which has a periodic behavior with respect to η is then provided. Section 3 is devoted to the study of an (n-1)-phase elliptic fiber embedded in an infinite medium under remote in-plane uniaxial, biaxial, and shear loadings. The aim of Section 4 is to estimate the effective in-plane shear modulus of nano-composites consisting of a distribution of either aligned or randomly oriented elliptic multi-coated fibers. In continue, for illustration of the current developments several descriptive examples, studying the effects of micropolar internal length scales, size of the elliptic coated fibers, volume fraction, shape, rigidity, and coating thickness on the effective in-plane shear modulus of the considered nanocomposite.

2. Formulation

Cosserat and Cosserat [38] assumed that each material point can have 3 translational as well as 3 rotational degrees of freedom. In other words, each point of a Cosserat medium has the degrees of freedom of a rigid body. The introduction of the concept of microinertia into Cosserat continuum results in micropolar theory of Eringen [39]. If the microrotation and macrorotation associated with a Cosserat continuum theory are set equal, then the theory reduces to the couple stress theory of Toupin [40], Mindlin and Tiersten [41], and Grioli [42]. This theory is often referred to as ”Cosserat theory with constrained rotation”. Since the current developments are based on micropolar theory, a brief introduction of the fundamentals, bearing some specific concepts and relations, are given in this section.

The kinematical considerations within micropolar theory lead to

$$\varepsilon_{ij} = u_{j,i} + e_{jik}\,\phi_k, \tag{1}$$

$$\kappa_{ij} = \phi_{i,j}, \tag{2}$$

where e_{ijk} is the permutation tensor, ε_{ij} is the strain, u_j is the displacement, ϕ_k is the microrotation, and κ_{ij} is the gradient of the microrotation. A comma ", " in the subscript denotes partial differentiation; e.g., $u_{j,i} \equiv \partial u_j/\partial x_i$. In this section, all the indices run over 1,2,3. It is noteworthy to mention that within micropolar theory the macrorotation, $\frac{1}{2}\, e_{ijk}\, u_{k,j}$ is not equal to the microrotation, ϕ_i. The equilibrium equations become

$$\sigma_{ji,j} + B_i = 0, \tag{3a}$$

$$m_{ji,j} + e_{ijk}\sigma_{jk} + C_i = 0, \tag{3b}$$

where σ_{ij} is the stress, B_i is the body force, m_{ij} is the surface moment normally referred to as the couple stress and C_i is the body couple per unit volume. Notice that as a consequence of including couple stresses and body couples, the stress tensor σ_{ij} is no longer symmetric. For an elastic isotropic micropolar material, the constitutive relations are given by

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + (\mu + k)\varepsilon_{ij} + \mu\,\varepsilon_{ji}, \tag{4a}$$

$$m_{ij} = \alpha\kappa_{ll}\delta_{ij} + \beta\kappa_{ij} + \gamma\kappa_{ji}, \tag{4b}$$

in which δ_{ij} is the Kronecker delta and λ, μ, k, α, β, γ are the micropolar elastic moduli. Note that classical elasticity relations correspond to the case where $k = \alpha = \beta = \gamma = 0$. The requirement that the strain energy function be positive definite puts the following restrictions on these moduli

$$\begin{gathered} 0 \le 3\lambda + 2\mu + k, \quad 0 \le 2\mu + k, \quad 0 \le k, \\ 0 \le 3\alpha + \beta + \gamma, \quad 0 \le \gamma + \beta, \quad 0 \le \gamma - \beta, \end{gathered} \tag{5}$$

In order to have unique values for the displacement and microrotation fields, certain relations must be satisfied among the deformation tensors. These are the integrability conditions for basic partial differential equations, which are known as the compatibility conditions [43].