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A cost-effective isogeometric approach for composite plates based on a stress recovery procedure

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ABSTRACT

This paper introduces a cost-effective strategy to simulate the behavior of laminated plates by means of isogeometric 3D solid elements. Exploiting the high continuity of spline functions and their properties, a proper out-of-plane stress state is recovered from a coarse displacement solution using a post-processing step based on the enforcement of equilibrium in strong form. Appealing results are obtained and the method is shown to be particularly effective on slender composite stacks with a large number of layers. These are indeed the cases where traditional (e.g., "layerwise") approaches are more computationally heavy and where researchers are more inclined to look for alternatives, making the proposed method a very attractive solution.

1. Introduction

Composite materials are used in a wide variety of fields such as aerospace or automotive. The study of composite laminates has become more and more important along the past years especially because of their light weight and very resistant mechanical properties. A composite laminate is usually made of several layers of highly resistant fibers embedded in soft matrix. Laminate structures tend to be prone to damage at the interfaces between layers, this mode of failure being referred to as delamination. The prediction and evaluation of damage in composite laminates demands an accurate evaluation of the three-dimensional stress state through the thickness, although most of the studies available in the literature consider the laminate as a two-dimensional object. Classical two-dimensional theories such as shell approaches are not accurate enough to reliably predict interlaminar damage and delamination. Accordingly, a number of layerwise theories have been developed to compute more accurately the mechanical state inside the laminate. Such methods often rely on heavy computations and hybrid approaches in order to be able to capture the complex behavior of the interlaminar interfaces. The numerical counterpart of such layerwise or hybrid theories was approached mostly using standard finite element discretization (see, e.g. [1,2], and references therein).

Over the last decade, many novel methods have been proposed to enhance the standard finite element framework. Among them, Isogeometric Analysis (IGA) is a concept proposed by Hughes et al. [3] where the shape functions used in computer aided design are also approximating physical fields and state variables. It thus relies on spline functions, like, e.g., NURBS (Non Uniform Rational B-Splines), to approximate both the geometry and the solution. In addition to improve the pipeline between design and analysis, spline shape functions possess properties which can be used to drastically improve the performance of numerical analysis compared to standard finite element method. For example, spline shape functions can provide easily high order approximation and highly simplify the refinement process. Moreover, their smoothness guarantees higher accuracy and opens the door to the discretization of high-order PDEs in primal form. IGA has been successfully used to tackle a large variety of problems, including solid and structures (see, e.g. [4–8]), fluids (see, e.g. [9,10]), fluid-structure interaction (see, e.g. [11,12]), and other fields (see e.g. [13,14]).

IGA methods have already been used to solve composite laminate problems. A wide variety of laminated models have been implemented and studied within the IGA framework, especially relying on high-order theories [15–18]. These techniques mostly utilize enhanced shell and plate theories. In addition to these 2D approaches, some methods can be used to compute the full 3D stress state using 3D isogeometric analysis such as in Refs. [18–20]. In such approaches, each ply of the

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laminate is delimited by a C^0-continuous interface. Such an approach (referred in the following as "layerwise") involves a large number of degrees of freedom when a lot of layers are composing the laminate. Thus it may be not completely satisfactory, despite its accuracy, due to its high cost.

This paper presents an approach consisting in using 3D computations with a reduced number (namely, one) of elements through the thickness and a layerwise integration rule, which is then post-processed in order to obtain an accurate 3D stress state. The post-processing relies on the integration of the equilibrium equations (such as in the recovery techniques proposed in Refs. [16,21–26]) to compute the stresses through the thickness from the in-plane ones. This allows to drastically reduce the computational time compared to the 3D layerwise approach as the number of degrees of freedom is significantly decreased. The post-processing operation is in fact very fast and its cost does not increase significantly with the number of degrees of freedom. Solutions obtained using the proposed technique are actually close to those provided by full "layerwise" 3D isogeometric analysis, at a fraction of the cost, in particular when many layers are present (which is indeed the case where layerwise approaches are extremely demanding in terms of computational cost, while accurate and inexpensive alternative strategies are desirable).

The effectiveness of the proposed approach relies on two ingredients, both granted by the peculiar properties of IGA, namely:

(i) The capability of obtaining accurate in-plane results with a coarse mesh with only one element through the thickness;
(ii) The higher continuity allowing an accurate computation of stresses and of their derivatives from displacements.

The structure of the paper is as follows. First, the considered isogeometric strategies are briefly introduced. Then, the 3D geometry and the mechanical problem used as a test case is detailed along with its analytical solution. Results using the two considered standard isogeometric approaches are then presented. Finally, the post-processing approach is proposed along with numerical results and a comparison with those provided by full "layerwise" 3D isogeometric analysis. Several cases using different numbers of quadrature points per material layer, numbers of layers and thicknesses are considered to show the effectiveness of the method.

2. Standard IGA strategies for 3D analysis of laminates

In this section we present a preliminary discussion on standard IGA strategies for 3D analysis of laminates. For the sake of conciseness we avoid reporting an introduction on the basic concepts of IGA for 3D elasticity, including basics of B-Splines, NURBS, etc. Readers are instead refered to [27] and references herein. Here, we just recall that we will consider a standard IGA consisting of an isoparametric Galerkin formulation where splines are used for approximating both geometry and displacements. As opposed to the framework typically used to compute laminate shell and plate solutions [1,16,28], we choose here to perform a full "layerwise" 3D computation. Such an approach has already been used in combination with IGA in order to solve multilayered plate problems [18–20] and along those lines we start considering two different sets of shape functions, as follows:

2.1. Layerwise approach

The first considered approach is similar to high order finite element methods. Each layer is in fact modeled by one patch through the thickness and C^0 continuity is kept between elements (see Fig. 1(a)). Such an approach is completely layerwise and, as a consequence, the number of degrees of freedom is proportional to the number of layers. A standard integration rule is adopted, namely $p+1$ points in each direction (p being the degree of the shape functions). This approach is equivalent to the one proposed in Refs. [18,20].

2.2. Single-element approach

The second approach actually uses a single element through the thickness (see Fig. 1(b)), strongly reducing the number of degrees of freedom with respect to the previously mentioned method. To account for the presence of the layers, a special integration rule is adopted, consisting of a q-point Gauss rule ($q \geq 1$) over each layer. This can be considered as a special homogenized approach with a layerwise integration. Such a method would *a priori* not give sufficiently good results (in terms of through-the-thickness stress description) but could be easily coupled with a post-processor to improve the solution, and this will be the object of Section 4.

It should be noted that the in-plane continuity of the shape functions in both approaches is the same.

3. Test case: the Pagano layered plate

The test case used in this study is the classical one proposed by Pagano [29]. It consists of a simply supported multilayered 3D plate with a sinusoidal loading on top and a loading-free bottom face (see Fig. 2). This problem can be easily parameterized which allows to analyze many cases in terms of layer number and distribution. In the following study, a few examples are considered using different numbers of layers (i.e., 3, 4, 11, and 34). In all these cases, the loading conditions are the same (namely, a two dimensional sinus with a period equal to twice the length of the plate), while the thickness of every single layer is set to 1 mm, and the length of the plate is chosen to be S times larger than the total thickness t of the laminate. Fig. 2 shows the summary of the test problem in the case of three layers.

The laminate is composed by orthotropic layers placed orthogonally on top of each other (thus creating a 90/0/90/ … laminate). For each layer, we have:

$$\boldsymbol{\sigma} = \mathbb{C}\boldsymbol{\varepsilon} \tag{1}$$

where the elasticity tensor $\mathbb{C}$ can be expressed, using Voigt notation, as:

$$\mathbb{C} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_2} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{31}}{E_3} & -\frac{\nu_{32}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}, \tag{2}$$

and the material parameters for these layers are the following:

$$E_2 = E_3 = \frac{E_1}{25}$$
$$G_{12} = G_{13} = \frac{G_{23}}{2.5}$$
$$\nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{3}$$

The pressure field used as the top boundary condition is

$$p(x, y) = \sigma_0 \sin\left(\frac{\pi x}{St}\right)\sin\left(\frac{\pi y}{St}\right) \tag{4}$$

In our test, we always select $E_1 = 25$ GPa, $G_{23} = 0.5$ GPa and $\sigma_0 = 1$.

3.1. Analytical results

Each stress component presents a different distribution along thickness. In particular, in-plane normal components of the stress tensor (i.e., σ_{11} and σ_{22}) are discontinuous along the thickness although the displacement solution is continuous in all cases. This is obviously due to the differences in term of material parameters between the layers. On