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Thermo-mechanical analysis of axially and transversally Function Graded Beam

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ABSTRACT

A generalized non-conventional finite element model for beam structure have been conducted, based on Timoshenko beam theory, this model is capable of static and dynamic analysis for both axially and transversally Function Graded Beam (FGB), its shape function not only depend on length but also depend on the material properties and cross-section geometry. A new convergence model for analysis of axially FGB has been developed, overcoming the problem of number of elements to length dependency in axial gradation, proposed model based on numerical integration by defining a local constant value of property for each element. Model validation for both transversally and axially FGB was done, by comparing the obtained results with published ones. Thermal analysis had been conducted for FGB operating in high temperature environment, with studying the effect of temperature change on the constituent material properties in both cases and its effect on dynamic response and natural frequencies. Concluding high accuracy model, its accuracy could reach high order shear deformation theory (HOSDT) in some applications, new convergence model for analysis of axially FGB, great influence of shear correction factor in Timoshenko beam and finally a good comparison between thermo-mechanical loading in axial and transversal FGBs.

1. Introduction

The main benefit of using functionally graded materials (FGMs) instead of conventional materials is that the internal composition of their component materials can be tailored to satisfy the requirements of a given structure with continues variation of properties. The internal structure of the material could be prepared to manufacture high temperature pressure vessels, thermal exposed structures and other mechanical or combined loaded structure type. Generally the ultimate goal is to help to determine if the structures constructed of functionally graded materials can be used instead of conventional materials in some applications. One of these applications is a space shuttle, where the aluminum substructure is shielded by a thermal protection system (TPS) barrier consisting of several layers of primers, tile, adhesives, fibers, and coatings which face problem of separation under excessive thermal loads, FGMs can be a good proposed solution for the separation problem. Additionally, mass could be minimized by tailoring the ingredient of each component based upon the load and stress interactions in different areas of the structure.

The extensive research in this field, which started with the pioneering work of Suresh and Mortensen [1], Reddy [2], and Sankar [3] has led to the development of several design approaches for analyzing FGM structures. Number of reviews dealing with various aspects of FGM have been published since 1990, some of these reviews using FGMs are Pindera et al. [4] in 1994 [5], in 1995 and in 1997, others deal generally about FGMs and its modeling Markworth et al., in 1995, S. Suresh et al., in 1998, Victor Birman et al. [6] in 2007 and Pankaj et al. [7] in 2014, then S. Suresh and A. Mortensen [8] in 1997, studied thermal stresses and thermo-mechanical behavior. Chakraborty et al. [9] in 2003, proposed a two-node beam element for FGMs based on FOSD theory applying it to static, thermal, free vibration and wave propagation problems, defining some limitation in FGB analysis and presenting non-conventional finite element model. Şimşek [10] in 2007, present free vibration analysis of FGBs by using a high order shear deformation (HOSDT) theory. Şimşek [11] in 2009, made static analysis of a uniformly distributed loaded FGB using Ritz method, also Şimşek [12] and [13] in 2010, The first, studied fundamental frequency analysis of FGBs by using different higher-order beam theories, the

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Nomenclature

E	Young's modulus
ν	Poisson's ratio
P	Material properties
P_t	property of the topmost layer
P_b	property of the bottommost layer
h	Thickness
x, z	Length and thickness coordinates
V_f	the volume fraction
n	Power law exponent
L	Beam or element length
u_0,w_0	Axial and transverse displacement
ϕ	Rotation displacement
ε	Normal strain
γ	Shear strain
K	Curvature
σ	Stress
Q	Stiffness element
G	Shear modulus of elasticity
k_s	shear correction factor
T	kinetic energy
ρ	Density
U	strain energy
$\tilde{Q}$	Generalized stiffness element
A_{11},B_{11},D_{11},A_{55}	Integrated stiffness coefficients
I_0,I_1,I_2	Integrated mass coefficients
c_1,c_2,…,c_{10}	Displacement coefficients
α_t,β_t	coupling between stiffness coefficients
$\varkappa(x)$	Shape function matrix
a	Displacement coefficients vector
$\hat{u}$	Nodal displacement vector
u	Element displacement vector
B	Curvature matrix
K	Stiffness matrix
M	Mass matrix
F	Nodal forces vector
P_b	element distributed load vector
$\alpha(z)$,$\alpha(x)$	coefficient of thermal expansion
ΔT	temperature change
AT_{11},BT_{11}	Thermal stiffness coefficients
R	thermal load vector
A	cross-section area
I	area moment of inertia
ψ	Non-dimensional parameter in shape function
$\overline{w}$	Non-dimensional deflection in z-direction
ω	Natural frequency
$\overline{\omega}$	Non-dimensional natural frequency
b	Beam width
L/h	length to thickness ratio
P_0, P_{-1}, P_1, P_2, P_3	Temperature dependent coefficients
T_0	room temperature

second deals with non-linear dynamic analysis of a FGB due to a moving harmonic load using Timoshenko beam theory with the von-Kármán's non-linear strain–displacement relationships, then Şimşek [14] in 2012, presented dynamic behavior of axially FGB under action of a moving harmonic load. Sankar [3] in 2001, presented elasticity solution for FGBs, also Sankar [15] in 2002, investigated thermal stresses on FGBs. X.-F. Li [16] in 2008, analyzed static and dynamic behaviors of Timoshenko FGB and Euler–Bernoulli FGB using unified approach. Yong Huang et al. [17] in 2010, introduced another approach for free vibration of axially FGBs with non-uniform cross-section. Amal E. Alshorbagy et al. [18] in 2011, studied free vibration characteristics of a FGB by finite element method (FEM), also E. Alshorbagy [19] in 2013, presented vibration characteristics of a FGB included temperature effects. J. Murin et al. [20] in 2013, presented effect of the shear correction function in the FGB modeling. Peter D.Dunning et al. [21] in 2014, dealed with aero-elastic tailoring of a plate wing with FGM. M. Filippi et al. [22] in 2014, introduced a static analyses of FGM beams by various theories and finite elements.

In the thermal analysis field, a thermal residual stresses from fabrication of FGM system was investigated one-dimensionally by Ravichandran [23] in 1995. Rao [24] in 2005, exploited global higher-order deformation theory for analysis of thermal buckling of plates made of FGMs. Lu et al. [25] in 2008, introduced semi-analytical elasticity solutions for thermal and static bending deformation of bi-directional FGBs using differential quadrature method (DQM). Shariyat [26] also in 2008, studied the effect of thermo-electro-mechanical loads on buckling of hybrid FGM cylindrical shells. Santos et al. [27] in 2008, analyzed transient thermal loading effect on thermoelastic FG cylindrical shells, Fraid et al. [28] in 2010, presented free vibration analysis of three-dimensional temperature dependent FGM curved panels using a hybrid semi-analytic DQM, Mahi et al. [29] in 2010, studied analytically the vibration of FGBs for temperature-material dependent. F. Tornabene et al. [30] in 2014, introduced different volume fraction distributions through-the-thickness, such as four-parameter power law, Weibull and exponential distribution. Also F. Tornabene et al. [31] in 2016, investigated several laminations schemes and various doubly-curved shells and panels reinforced by carbon nanotubes (CNTs) using exponential distributions along the thickness of the structures. N. Fantuzzi et al. [32] in 2016, studied free vibration of arbitrary shape cracked functionally graded plates using four-parameter volume fraction.

In this work, trying to solve the arising question that is how to implement element formulations for structures composed of axially or transversally FGMs, first order shear deformable (Timoshenko) beam model is investigated at first, then applied to beams subjected to spatial variations in temperature combined with mechanical loading, studying stress distribution and the effect of temperature change on the material properties and dynamic response for both axially and transversally gradation techniques.

2. Mathematical formulation

2.1. Material gradation and temperature dependency

In this study, it is assumed that the beam is made of ceramics and metal, and the effective material properties of FG beam such as, Young's modulus (E), Poisson's ratio (υ), shear modulus (G) and mass density (ρ) vary continuously according to power-law through the thickness and axial direction, in addition to their variation for temperature.

According to power law distribution P-FGM, the material properties at a specified coordinate can be expressed as Alshorbagy et al. [18]:

- For gradation through thickness direction