Composites Part B 99 (2016) 48–62

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Fast analysis of non-symmetric panels using semi-analytical techniques

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A R T I C L E I N F O

Article history:
Received 26 February 2016
Received in revised form
26 April 2016
Accepted 17 May 2016
Available online 26 May 2016

Keywords:
Plates
Laminates
Buckling
Analytical modeling
Postbuckling

A B S T R A C T

A semi-analytical approach is presented for the analysis and optimization of laminated panels with non-symmetric lay-ups, and with the possibility of introducing requirements on the buckling load, the postbuckling response and the eigenfrequencies. The design strategy relies on the combined use of semi-analytical techniques for the structural analysis and genetic algorithms for the optimization. The structural analysis is performed with a highly efficient code based on thin plate theory, where the problem is formulated in terms of Airy stress function and out of plane displacement, expanded using trigonometric series. Eigenvalue analyses are performed to determine eigenfrequencies and buckling load, while an arc-length method is adopted for the postbuckling computation. The genetic algorithm is implemented with proper alphabet cardinalities to handle different steps for the angles of orientation, while specific mutation operators are used to guarantee good reliability of the optimization. To show the potentialities of the proposed optimization toolbox, two examples are presented regarding the design of balanced non-symmetric laminates subjected to linear and nonlinear constraints. The accuracy of the semi-analytical predictions is demonstrated by comparison with finite element results and benchmark cases taken from the literature.

1. Introduction

In the past years, many efforts were directed towards the development of analytical and semi-analytical methods for the fast analysis of composite panels [1–3]. In most cases, the methods focused on symmetrically layered structures, thus avoiding the coupling between the in plane and out of plane behaviour of the panel. Relatively few works dealt with non-symmetric lay-ups. Similarly, several design optimization procedure were developed by restricting the design space to the case of symmetric lay-ups. Examples are found in two works of the authors [4,5], where analytical tools are coupled with genetic algorithms, and symmetric lay-ups are assumed. In order to fully exploit the tailoring opportunities offered by composite materials, novel analysis tools are needed to handle more generic lay-up configurations. In this context, closed-form solution are a useful mean to guarantee computational effectiveness, which is particularly useful when dealing with optimization procedures. However, the complexity of the mechanical couplings characterizing the response of generally layered panels often requires the introduction of several simplifying assumptions. An early work of Chandra [6] presents a single-term solution to analyse non-symmetric panels and is restricted to the case of axially compressed cross-ply configurations. The Ritz method is adopted by Dano and Hyer [7] to study the response of non-symmetric panels during the cooling from the cure temperature. More recently, closed-form solutions were derived by Diaconu and Weaver [8] using a single-term approximation to represent the out of plane displacement and considering compression load. Nie and Liu [9] extended the formulation to account for shear loads and elastic restraints. In both in Refs. [8,9], the approach is valid for infinitely long panels only, and any mode change or snap through cannot be accounted for. Many of the restrictions necessary to derive closed-form solution can be relaxed by adopting multiple-series solutions. In these cases, the equations can be obtained analytically, but the solution is computed numerically. An example is found in the work of Zhang and Matthews [10], where the Airy stress function and the out of plane displacement are approximated with sine terms or beam eigenfunctions. Zhang at al. [11] proposed

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http://dx.doi.org/10.1016/j.compositesb.2016.05.044
1359-8368/

a formulation based on Karman-Reissner plate theory and asymptotic series solution to study the buckling and the postbuckling response of non-symmetric plates. In both cases, the governing equations regard the out of plane equilibrium and the strain compatibility. The total number of degrees of freedom is still smaller if compared to finite elements, but significantly higher with respect to closed-form solutions. The present work presents an efficient semi-analytical approach based on multiple degrees of freedom, able to capture mode-jumping phenomena, and characterized by analysis time comparable to closed-form solutions. The approach, which is developed for non-symmetric plates with finite length and loading conditions of compression and shear, is adopted in the context of a design optimization based on genetic algorithms with linear and nonlinear constraints.

2. Semi-analytical model

The analysis tool is implemented in an efficient Matlab program and is developed for the analysis of flat plates layered with symmetric and non-symmetric stacking sequences. Loading conditions of compression and shear are accounted for.

Differently from the vast majority of papers dealing with plate analysis, the present approach does not consider the plate as a self-standing unit, but as a part of a larger structure. This is the typical situation observed in stringer- and frame-stiffened panels, where the overall structure – the fuselage of an aircraft, for instance – is obtained by the repetition of several, self-similar, structural elements. In many cases, it is still possible to isolate the single plate, but for a non-symmetric panel the behaviour can be significantly different.

A sketch of the structure under investigation is provided in Fig. 1(a). The model, which is here used for the linear buckling analysis, is composed of a central plate, highlighted in gray, surrounded by to half-bays along the longitudinal and the transverse direction. The central plate element has dimensions a × b, while the dimension of the overall repeating unit is twice the dimension of the central plate, i.e. 2a × 2b.

It is assumed that the structure undergoes local phenomena, meaning that the buckling modes and the postbuckling deformed pattern are characterized by null out of plane deflections along the boundaries, indicated in the figure as nodal lines, of the different plate elements composing the unit. This assumption stems from the consideration that, in a real structure, stringer and frames would be generally designed to avoid the onset of global deflections. Considering the unit of Fig. 1(a) as part of a larger structure, periodic conditions are imposed along the four outer boundaries. More specifically, it is assumed that the rotations and displacements along the outer left edge are equal to those of the right edge, and similarly for the upper and lower edge. Regarding the in plane displacements, the panel is free to expand or contract in both the directions, so that no induced stresses are introduced during the deformation process.

In the context of the postlbuckling analysis, the model can be further simplified, as illustrated in Fig. 1(b), where only the central plate is accounted for, and the effect of the surrounding structure is recovered by means of equivalent springs.

Thin plate assumption is introduced, and Classical Lamination Theory (CLT) is applied. Referring to the central plate element, a Cartesian coordinate system is taken over the panel midsurface with the x-axis directed along the longitudinal direction, the y-axis along the transverse direction and the z-axis to define a right-handed system. The laminate is layered with an arbitrary number of plies, not necessarily stacked to guarantee the symmetry with respect to the midplane of the panel. The only assumption here introduced is that of balanced laminate, meaning that the presence of a ply at $+\theta$ requires the presence of a ply at $-\theta$. The semi-inverse constitutive law of the laminate is:

$$\begin{Bmatrix} \boldsymbol{\xi} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{e} \\ -\mathbf{e}^T & \mathbf{d} \end{bmatrix} \begin{Bmatrix} \mathbf{N} \\ \mathbf{k} \end{Bmatrix} \tag{1}$$

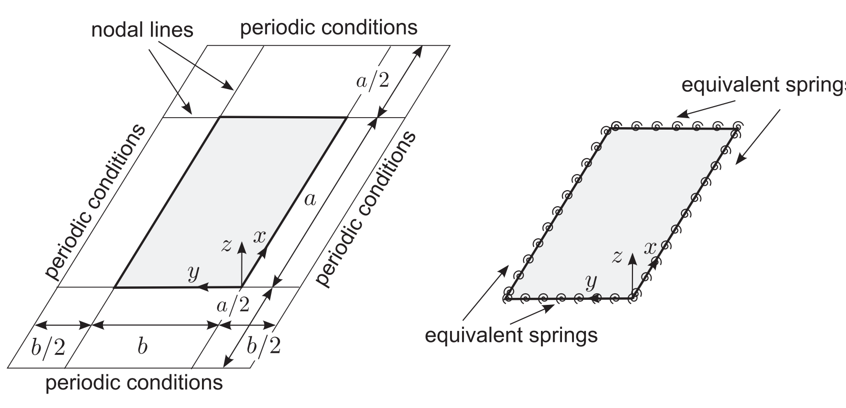
where **N** and **M** are the force and moment resultant along the thickness, while ξ and **k** are the membrane strains and the curvatures, respectively. The matrices **a, d, e** of Eq. (1) are related to the **A, B, D** matrices of CLT through the relations:

$$\mathbf{a} = \mathbf{A}^{-1} \quad \mathbf{e} = -\mathbf{A}^{-1}\mathbf{B} \quad \mathbf{d} = \mathbf{D} - \mathbf{B}\mathbf{A}^{-1}\mathbf{B} \tag{2}$$

Under the assumption of balanced laminate and using the Voigt notation, the only null terms of Eq. (1) are a_{16} and a_{26}.

2.1. Simplified buckling analysis

In the context of the analysis approach here described, the buckling analysis is conducted for two reasons. Firstly, to obtain an estimate of the buckling load of the structure, and secondly to determine the amount of restraint provided by the surrounding structure to the central panel. As illustrated next, the portion of structure that surrounds the central panel of Fig. 1(a) can be condensed to a set of equivalent torsion springs along the plate boundaries. This model reduction allows to perform the



(a) (b)

Fig. 1. Panel geometry and reference system: (a) representative unit used for linear buckling analysis, (b) reduced model used for postbuckling analysis.