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A hierarchical formulation of the state-space Levy's method for vibration analysis of thin and thick multilayered shells

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ABSTRACT

The state-space approach in conjunction with the Levy's method is used to solve exactly the free vibration problem of specially orthotropic multilayered cylindrical and spherical panels. A hierarchical formulation is presented to build the matrices of the method from small elementary blocks which are invariant with respect to the order and typology of the kinematic shell theory. As a result, the analytical effort to derive the governing equations is minimized and a large number of Levy-type solution based on low to high order, equivalent single-layer or layerwise theories, can be generated within the same mathematical framework. Thereby, the refinement of the two-dimensional shell model can be tailored according to the thickness ratio and the degree of anisotropy of the problem under study and the desired accuracy. Some illustrative results on both thin and thick, laminated and sandwich panels with various boundary conditions are presented and discussed to show the potential of the formulation.

1. Introduction

The so-called Levy's method is a well established technique aimed at obtaining exact bending, buckling and vibration solutions of particular plate and shell problems. The origin and name of the method are commonly ascribed to the seminal work of Maurice Levy, who successfully solved in 1899 the bending problem of thin isotropic rectangular plates with simply supported two opposite edges and arbitrary conditions of supports on the two remaining opposite edges using single Fourier series [1]. As observed by Leissa [2], the same type of solution was actually first used by Voigt in 1893 to determine the transverse vibrations of rectangular plates [3]. In spite of this, the single trigonometric series expansion is now conventionally referred to as Levy's solution for both static and dynamic problems [4].

The availability of exact solutions for some plate and shell problems is valuable as they serve as important references for checking the convergence and accuracy of approximate and numerical methods. To this aim, the Levy's method, even if at the cost of greater complexity, is more general and of practical interest than the Navier's method, which is restricted to exact analysis of plates and shells with all edges simply supported. However, it does not have an entirely general character and shares some limitations with the Navier's method, since both can be applied only to particular geometries (i.e., rectangular plates, cylindrical and spherical shells) and material symmetries (i.e., specially orthotropic structures), for which an exact solution of the corresponding boundary-value problem is viable.

This paper is focused on the application of the Levy's method to vibration problems, which has a long and successful history, especially for plates [2,5–24]. In particular, the present work is aimed at presenting an advanced state-space formulation of the method for two-dimensional (2-D) exact vibration analysis of cylindrical and spherical single- and multi-layered specially orthotropic panels having both small and large thickness and shallowness ratios. It is worth noting that, contrary to the fairly large number of papers on plates, very few works are available in the open literature on Levy-type vibration solutions of shells [25–29]. The main novelty of the present contribution relies on a versatile hierarchical technique to build the final matrices of the state-space Levy's method from elementary blocks, called fundamental nuclei, which are invariant with respect to the 2-D kinematic shell theories. In so doing, the tedious and cumbersome analytical effort required for deriving the governing equations related to each specific theory is avoided and a large family of Levy-type vibration solutions of curved panels based on kinematic theories of different order and typology can be automatically

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generated within the same mathematical framework. The methodology stems from the powerful technique developed by Carrera [30] and makes extensive use of indicial notation. The application of the Carrera's formulation to the Levy's method was originally proposed by the present author for laminated and FGM plates [31,32] and recently applied by Rezaei and Saidi for vibration analysis of thick porous-cellular plates [33]. In this work, the method is generalized and extended to curved panels.

As shown later, the hierarchical nature of the present formulation allows the accurate exact vibration analysis of both thin and thick, deep and shallow multilayered shell structures. Indeed, an exact 2-D analysis of a multilayered shell does not imply that the corresponding results are also accurate compared to a truly three-dimensional (3-D) analysis. It is known that the thickness ratio (defined as the ratio between the thickness of the panel to the shortest of the span lengths or radii of curvature) and the shallowness ratio (defined as the ratio of the shortest span length to one of the radii of curvature) are two important parameters governing the choice of an appropriate 2-D kinematic model of the curved panel having a desired accuracy [34]. Classical low-order theories are typically employed when the panel is thin and shallow, whereas refined higher-order 2-D shell theories are required to achieve a satisfactory accuracy for thick and deep shells. The accuracy is also largely affected by the frequency range of interest and the degree of anisotropy in the thickness direction [35]. Broadly speaking, for a fixed kinematic theory it usually degrades as the wavelength of the vibration mode is of the order of magnitude of the panel thickness and as the variation of mechanical properties through the thickness direction increases like the case of sandwich panels. By means of the present formulation, the refinement of the shell model can be tailored on the specific case under investigation and the accuracy of the refined model can benefit from the exactness of the Levy-type solution, without being adversely influenced by the convergence and stability properties of an approximate method.

The paper is organized as follows. After some preliminary definitions in Section 2 and a concise presentation in Section 3 of the family of 2-D shell theories employed in this work, the equations of motion and related boundary conditions of cylindrical and spherical panels are presented in Section 4 according to the compact indicial form introduced by Carrera [30]. The hierarchical construction of the matrices involved in the Levy-type solutions from small invariant elementary blocks is detailed in Section 5. Some illustrative vibration results based on shell theories of different order and typology are shown in Section 6 along with comparison with exact 3-D analysis and other 2-D approaches. Finally, Section 7 contains some concluding remarks.

2. Preliminaries

Let's consider the cylindrical and spherical multilayered panels in Fig. 1, which are composed of N_ℓ layers of homogeneous orthotropic material. Each layer k has thickness h_k and is numbered sequentially from bottom ($k=1$) to top ($k=N_\ell$) of the panel. The total thickness of the panel is $h=\sum_{k=1}^{N_\ell} h_k$. The undeformed middle surface Ω_k of each layer is described by the two orthogonal curvilinear coordinates α and β. Let z_k denote the rectilinear local thickness coordinate in the normal direction with respect to Ω_k. The components of the displacement field $\mathbf{u}^k$ of the k-th layer are indicated as u_α^k, u_β^k and u_z^k in the α, β and z directions, respectively.

According to 3-D elasticity and considering curved panels with constant curvature, the in-plane strains $\boldsymbol{\varepsilon}_\mathrm{p}^k = \left\{ \varepsilon_{\alpha\alpha}^k \quad \varepsilon_{\beta\beta}^k \quad \gamma_{\alpha\beta}^k \right\}^\mathrm{T}$ of layer k can be expressed as a function of the displacement components $\mathbf{u}^k = \left\{ u_\alpha^k \quad u_\beta^k \quad u_z^k \right\}^\mathrm{T}$ by the following relation:

$$\boldsymbol{\varepsilon}_\mathrm{p}^k = \left(\mathcal{D}_\mathrm{p}^k + \mathcal{A}_\mathrm{p}^k\right)\mathbf{u}^k \tag{1}$$

where

$$\mathcal{D}_\mathrm{p}^k = \begin{bmatrix} \frac{1}{H_\alpha^k}\frac{\partial}{\partial\alpha} & 0 & 0 \\ 0 & \frac{1}{H_\beta^k}\frac{\partial}{\partial\beta} & 0 \\ \frac{1}{H_\beta^k}\frac{\partial}{\partial\beta} & \frac{1}{H_\alpha^k}\frac{\partial}{\partial\alpha} & 0 \end{bmatrix} \qquad \mathcal{A}_\mathrm{p}^k = \begin{bmatrix} 0 & 0 & \frac{1}{H_\alpha^k R_\alpha^k} \\ 0 & 0 & \frac{1}{H_\beta^k R_\beta^k} \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

R_α^k and R_β^k are the curvature radii of the α and β coordinate curves, respectively, at the generic point of the middle surface Ω_k of the layer, and

$$H_\alpha^k = 1 + \frac{z_k}{R_\alpha^k} \qquad H_\beta^k = 1 + \frac{z_k}{R_\beta^k} \tag{3}$$

It is noted that $H_\alpha^k = 1$ for cylindrical panels since $R_\alpha^k = \infty$ and $H_\alpha^k = H_\beta^k$ for spherical panels since $R_\alpha^k = R_\beta^k$. Note also that when $1/R_\alpha^k = 1/R_\beta^k = 0$, the above relations degenerate to those for plates.

Similarly, the normal strain components $\boldsymbol{\varepsilon}_\mathrm{n}^k = \left\{ \gamma_{\alpha z}^k \quad \gamma_{\beta z}^k \quad \varepsilon_{zz}^k \right\}^\mathrm{T}$ can be expressed as follows

$$\boldsymbol{\varepsilon}_\mathrm{n}^k = \left(\mathcal{D}_\mathrm{n}^k - \mathcal{A}_\mathrm{n}^k + \mathcal{D}_z^k\right)\mathbf{u}^k \tag{4}$$

where

$$\mathcal{D}_\mathrm{n}^k = \begin{bmatrix} 0 & 0 & \frac{1}{H_\alpha^k}\frac{\partial}{\partial\alpha} \\ 0 & 0 & \frac{1}{H_\beta^k}\frac{\partial}{\partial\beta} \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{A}_\mathrm{n}^k = \begin{bmatrix} \frac{1}{H_\alpha^k R_\alpha^k} & 0 & 0 \\ 0 & \frac{1}{H_\beta^k R_\beta^k} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

and $\mathcal{D}_z^k = \frac{\partial}{\partial z}\mathbf{I}_3$.

Assuming a linearly elastic orthotropic material, the constitutive equations of the k-th layer in the laminate reference coordinate system are written as

$$\begin{aligned} \boldsymbol{\sigma}_\mathrm{p}^k &= \tilde{\mathbf{C}}_\mathrm{pp}^k \boldsymbol{\varepsilon}_\mathrm{p}^k + \tilde{\mathbf{C}}_\mathrm{pn}^k \boldsymbol{\varepsilon}_\mathrm{n}^k \\ \boldsymbol{\sigma}_\mathrm{n}^k &= \tilde{\mathbf{C}}_\mathrm{pn}^{k^\mathrm{T}} \boldsymbol{\varepsilon}_\mathrm{p}^k + \tilde{\mathbf{C}}_\mathrm{nn}^k \boldsymbol{\varepsilon}_\mathrm{n}^k \end{aligned} \tag{6}$$

where $\boldsymbol{\sigma}_\mathrm{p}^k = \left\{ \sigma_{\alpha\alpha}^k \quad \sigma_{\beta\beta}^k \quad \tau_{\alpha\beta}^k \right\}^\mathrm{T}$ is the vector of in-plane stresses, $\boldsymbol{\sigma}_\mathrm{n}^k = \left\{ \tau_{\alpha z}^k \quad \tau_{\beta z}^k \quad \sigma_{zz}^k \right\}^\mathrm{T}$ is the vector of normal stresses, and the matrices of stiffness coefficients given by

$$\tilde{\mathbf{C}}_\mathrm{pp}^k = \begin{bmatrix} \tilde{C}_{11}^k & \tilde{C}_{12}^k & \tilde{C}_{16}^k \\ \tilde{C}_{12}^k & \tilde{C}_{22}^k & \tilde{C}_{26}^k \\ \tilde{C}_{16}^k & \tilde{C}_{26}^k & \tilde{C}_{66}^k \end{bmatrix} \qquad \tilde{\mathbf{C}}_\mathrm{pn}^k = \begin{bmatrix} 0 & 0 & \tilde{C}_{13}^k \\ 0 & 0 & \tilde{C}_{23}^k \\ 0 & 0 & \tilde{C}_{36}^k \end{bmatrix} \qquad \tilde{\mathbf{C}}_\mathrm{nn}^k = \begin{bmatrix} \tilde{C}_{55}^k & \tilde{C}_{45}^k & 0 \\ \tilde{C}_{45}^k & \tilde{C}_{44}^k & 0 \\ 0 & 0 & \tilde{C}_{33}^k \end{bmatrix} \tag{7}$$

are derived from those expressed in the layer reference system