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# An analytical model on through-thickness stresses and warpage of composite laminates due to tool–part interaction

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ABSTRACT

Tool–part interactions play an important role in the deformation of composite components during forming, especially for thin laminates. In this paper, taking into account the effect of slipping between the tool and composite laminates, an analytical model is proposed to predict the through-thickness residual stresses and warpage of a composite part during the curing process without extensive resin characterization. A parametric study examining the effects of various interfacial shear stresses on warpage is performed and the results are compared with experimental data in the literature. Good agreements are obtained. The results show that warpage is enhanced linearly with the increase of interfacial shear stress and the stress distribution does not take on an exponential form for $[90/0]_s$ lay-up. It is also shown that this model could successfully predict part processing deformations without extensive resin characterization.



## 1. Introduction

Autoclave assisting cure of fiber-reinforced polymers using prepregs is an important manufacturing method to fabricate high performance composite parts. In this process, a composite laminate is laid up on a tool surface and then covered with bagging materials such as bleeder layers, breather cloths or vacuum bags. And then, the entire assembly is subjected to a controlled cycle of temperature and pressure in an autoclave. Residual stresses invariably arise during this process and often cause the formed composite part to distort from the shape of the tool from which it is laid up on removal of the tool. This distortion is known as 'warpage' or 'spring-in'. A variety of factors affect the magnitude and distribution of residual stress, such as tool–part interaction, curing shrinkage of resin, properties of each ply including thermal expansion coefficient, pressure cycles applied during the fabrication process and non-uniform cooling of laminate due to severe temperature gradients [1–6]. Residual stress during fabrication greatly decreases the fatigue life and dimensional accuracy of composite parts [7]. Therefore, it is important to evaluate the magnitude and distribution of residual stress, as well as predict its effects on the performance of composite parts.

In terms of the resources, residual stresses in composites are classified into two categories: micro and macro residual stresses. The micro stress is in a single fiber–matrix composite and mainly results from the mismatch in thermal expansion coefficient and Young's modulus between fiber and matrix. On the macro-mechanical level, if the material is not stacked symmetrically with respect to the mid-plane, then during cooling down from high curing temperature, macro residual stress will generate in the laminates, which would further result in bending moments and warpage. However, stresses caused by tool–part interaction due to the mismatch of Coefficient of Thermal Expansion (CTE) between the tool and part should not be neglected [8–11], which is focused in this study.

Usually the tool material has much higher CTE than the composite. When the tool and composite part are forced together due to autoclave pressure and subjected to a temperature ramp, a shear interaction between the tool and the part along their interface will arise and place the laminate in tension, as shown in Fig. 1. Curing composite part has a very low shear modulus, therefore, plies close to the tool are stretched more than the plies further away, which will create a stress gradient through the thickness of the laminate. This non-uniform stress gradient locks in as the material cures upon removal of the tool, causes the composite part to warp. That is

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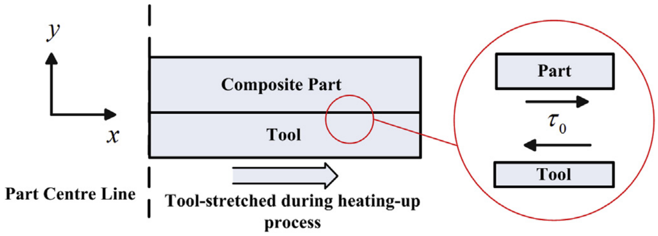


**Fig. 1.** Schematic of a composite part under sliding friction conditions due to tool expansion.

the reason for thin balanced laminates fabricated on flat tool to exhibit warpage after curing. It has been noted that, besides CTE mismatch, this stress gradient is also affected by other parameters such as the magnitude of autoclave pressure and tool surface condition.

The processing induced warpage of composite part has been studied over past 25 years. A number of predictive techniques that consider warpage effects have been presented in the literatures. Finite element method (FEM) is now a widely matured and increasingly accepted technique for modelling temperature, residual stress and warpage of composite part [12–14]. However, this approach often requires extensive experimental and material characterization and numerical processing capability. In contrast, closed form analytical solutions have the advantage for understanding the mechanisms of the residual stress and warpage. The classical lamination theory (CLT) and energy method solved in closed form solutions have been widely used to predict the shape of asymmetric laminates after the curing process [15–19]. However, tool–part interaction is the main reason for symmetric laminates to induce warpage after curing. Consequently, the main objective of the current work is to develop an analytical model to investigate the through-thickness stress and warpage due to tool–part interaction and improve the predictive capability of closed form solutions without considering the complications associated with cure kinetics and nonlinear material characterization. With the use of experimental data, the interfacial shear stress between the tool–part is calibrated to an appropriate value and reasonable predictions could be made for other parts.

## 2. Interaction between tooling and composite part

A composite part is stretched due to the expansion of the tool during heating-up and it can be simplified to a classical beam undergoing tractions at the top and bottom surfaces as shown in Fig. 2. The stress and displacement can be written in the following form [10]:

$$
\begin{aligned}
u_{xx} &= \sum_{n=1}^{N}\left\{\sin(k_n x)\left(A_n e^{\beta_n y} + B_n e^{-\beta_n y}\right)\right\} + \varepsilon_{therm} x \\
\sigma_{xx} &= \sum_{n=1}^{N}\left\{E_{xx} k_n \cos(k_n x)\left(A_n e^{\beta_n y} + B_n e^{-\beta_n y}\right)\right\} \\
\tau_{xy} &= \sum_{n=1}^{N}\left\{G_{xy}\beta_n \sin(k_n x)\left(A_n e^{\beta_n y} - B_n e^{-\beta_n y}\right)\right\}
\end{aligned}
\tag{1}
$$

where $\beta_n = ck_n$, $c = \sqrt{E_{xx}/G_{xy}}$, $k_n = (2n-1)\pi/l$, $n = 1,2,3,...,$ $u_{xx}$ and $\sigma_{xx}$ are the beam axial displacement and stress, respectively, and $\tau_{xy}$ is the shear stress, $E_{xx}$ and $G_{xy}$ are the beam axial elastic modulus and transverse shear modulus, respectively, $\varepsilon_{therm}$ is the axial free thermal strain, $l$ is the length of the laminate in the $x$ direction. $A_n$ and $B_n$ are unknowns and can be calculated by applying the boundary conditions at the bottom and top surfaces ($\tau_{xy} = \tau_b$ at $y = 0$ and $\tau_{xy} = \tau_t$ at $y = t$), as shown in Fig. 2.

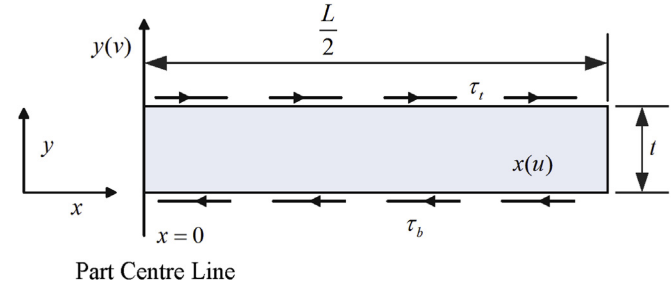


**Fig. 2.** Schematic of a beam under applied shear tractions on the top and bottom surfaces.

The composite part is made of multi-layers of laminate with different fiber orientations, as shown in Fig. 3. From Eq. (1), axial displacement and stress variation in the thickness direction can be given by:

$$
\begin{aligned}
u_{xx}^i &= \sum_{n=1}^{N}\left\{\sin(k_n x)\left(A_n^i e^{\beta_n^i y} + B_n^i e^{-\beta_n^i y}\right)\right\} + \varepsilon_{ther}^i x \\
\sigma_{xx}^i &= \sum_{n=1}^{N}\left\{E_{xx}^i k_n \cos(k_n x)\left(A_n^i e^{\beta_n^i y} + B_n^i e^{-\beta_n^i y}\right)\right\} \\
\tau_{xy}^i &= \sum_{n=1}^{N}\left\{G_{xy}^i \beta_n^i \sin(k_n x)\left(A_n^i e^{\beta_n^i y} - B_n^i e^{-\beta_n^i y}\right)\right\}
\end{aligned}
\tag{2}
$$

where a superscript of $i$ denotes the coefficients connected with the $i$-th layer; $A_n^i$ and $B_n^i$ are unknowns associated with the $i$-th layer. For $m$ layers, there are $2m$ unknowns which can be determined from the equilibrium and continuity conditions on the interface between the layers.

The layer axial displacement and shear stress should be equal on the interface between the two neighbor plies, which means that at a generic $i$-th interface ($y = y_i$), these are:

$$u_{xx}^i = u_{xx}^{i+1} \text{ and } \tau_{xy}^i = \tau_{xy}^{i+1} \tag{3}$$

which result in the following two equations:

$$
\begin{aligned}
&\sum_{n=1}^{N}\left\{\sin(k_n x)\left(A_n^i e^{\beta_n^i y_i} + B_n^i e^{-\beta_n^i y_i}\right)\right\} + \varepsilon_{ther}^i x \\
&\quad = \sum_{n=1}^{N}\left\{\sin(k_n x)\left(A_n^{i+1} e^{\beta_n^{i+1} y_i} + B_n^{i+1} e^{-\beta_n^{i+1} y_i}\right)\right\} + \varepsilon_{ther}^{i+1} x
\end{aligned}
\tag{4}
$$

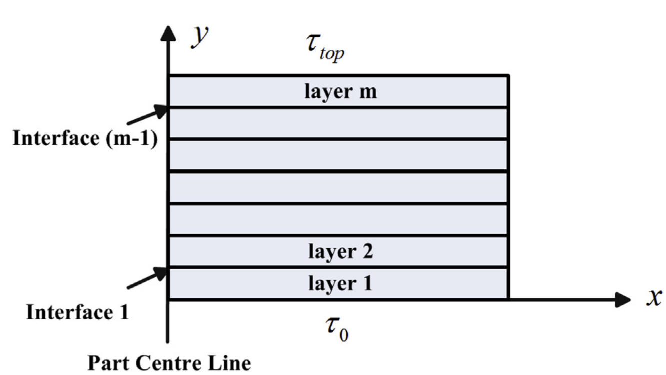


**Fig. 3.** Model of the laminate.