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Study on the effect of small scale on the wave reflection in carbon nanotubes using nonlocal Timoshenko beam theory and wave propagation approach

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ABSTRACT

This paper presents the wave propagation approach for analyzing the free vibration and wave reflection in carbon nanotubes. The propagation and reflection matrices obtained by the wave propagation method are useful for the analysis of mechanical energy transmission and reflection in micro/nano devices. These matrices can also be used for obtaining the natural frequencies of carbon nanotubes. Firstly, the obtained natural frequencies by this method are compared with the results in the literature. Then, the effect of small scale on the reflected power of an incident wave upon different boundary conditions is studied in details. It was found that the reflected power of an incident wave upon a simply supported boundary is independent of the small scale, incident wave frequency, material and geometrical parameters of the carbon nanotubes while the reflected power of a wave upon clamped and free boundary conditions depends on the small scale, incident wave frequency, material and geometrical parameters.

1. Introduction

Nanotechnology is one of the new fields of scientific researches which has drawn attention of so many researchers owing to superiority of nanoscale materials as mechanical, thermal, electrical and optical properties. Two allotropes of carbon, carbon nanotubes (CNTs) and graphene sheets (GSs), are the main nanoscale geometries which engineers attempt to extol their products by using such nanoscale materials. Of course, they should analyze the behavior of such materials using reliable mathematical methods. In general, computational tools in nanoscale can be categorized in three groups: atomistic simulations, continuum mechanics and hybrid atomistic-continuum. Atomistic and semi-atomistic methods model intra-molecular forces using some potential functions which are applicable for analysis with few numbers of atoms in short interval time. In fact, for large atomic structures they are so time consuming. Among these groups, the continuum mechanics is a reliable, simple and computationally non-expensive method compared to the other two. The main assumption in the classical continuum mechanics is that the stress at a point is a function of strain at that point. But, in the nanoscale, the spaces in the molecular lattices are comparable with the dimensions of such structures. So, the continuum theories need some appropriate changes to consider the size effects. In the 90's, some models were introduced by Mindlin and Eshel [1], Green and Rivlin [2] and Mindlin [3,4] to consider this effect on continuum model, but they are so complex due to their multiplicity of unknowns. Among them, the nonlocal elasticity theory which was introduced by Eringen [5] is the simplest one and it is being used for analyzing the mechanical behavior of nanostructures, especially the carbon nanotubes (CNTs). Many researchers have modeled the carbon nanotubes as elastic beams such as Timoshenko, Euler-Bernoulli, shells and elastic rod [6–10]. Reddy [11] presented analytical and numerical solutions on static deflections, buckling loads, and natural frequencies of three kinds of nonlocal beam theories known as Euler–Bernoulli, Timoshenko, Reddy and Levinson. Moreover, a nonlocal nonlinear formulation was derived by Reddy [12] for bending of classical and shear deformation theories of beams and plates. Thai [13] proposed a nonlocal shear deformation theory for bending, buckling, and vibration of nanobeams. Based on nonlocal continuum mechanics, Wang et al. [14] analyzed the free vibration of Euler–Bernoulli and Timoshenko nanobeams. Loya et al. [15]

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investigated the free transverse vibration of cracked Euler–Bernoulli nanobeams using nonlocal elasticity model. The vibration of double and multi-walled carbon nanotubes has also been investigated by Zhang et al. [16] and Chowdhury et al. [17], respectively. Based on nonlocal Timoshenko beam theory, Ansari and Ramezannezhad [18] analyzed the large amplitude vibrations of embedded multi-walled carbon nanotubes including thermal effects. In similar works, the nonlocal Timoshenko beam theory was used by Yang et al. [19] and Ke et al. [20] for nonlinear free vibration analysis of single- and embedded double-walled carbon nanotubes, respectively. Murmu and Adhikari [21] presented an analytical method for analyzing transverse vibration of double-nanobeam systems using nonlocal elasticity theory.

Various methods have been integrated with nonlocal elasticity theory to analyze nanostructures. Finite element method was used by Eltaher et al. [22,23] to study the free vibration and buckling analyses of functionally graded Euler–Bernoulli nanobeams. Moreover, Phadikar and Pradhan [24] studied nanobeams and nanoplates with a linear nonlocal formulation using finite element method. Zhang et al. [25] analyzed the bending, buckling, and vibration of micro/nanobeams using a hybrid nonlocal Euler–Bernoulli beam model. Furthermore, based on Euler–Bernoulli beam theory, Civalek and Akgoz [26] studied free vibration of microtubules via Differential Quadrature (DQ) method.

In classical method, the natural frequencies can be obtained by applying the boundary conditions to the general solution of differential equation of motion. There is an alternative method known as wave propagation method in which the vibrations are described as propagating waves traveling in the structure. This method is a simple, non-iterative and efficient method for calculation of the natural frequencies of the structures. This method was mainly used in macrostructures to study the free vibrations of Euler Bernoulli beams [27], and Timoshenko beams [28]. Argento and Scott [29] used the wave propagation method to analyze the vibration of a rotating Timoshenko shaft. In another work, Tan and Kang [30] investigated the wave motions in an axially strained, rotating Timoshenko shaft. The wave propagation in non-uniform waveguides such as non-uniform bars and non-uniform Euler Bernoulli beams was considered by Lee et al. [31]. Also, the wave method was used for free and forced vibrations of axially loaded cracked Timoshenko beams [32] and curved beams [33]. Mei [34] presented an exact wave-based analytical solution for obtaining the natural frequencies of classical planar frame structures, in which the coupling effect between bending and longitudinal vibrations was taken into account. Moreover, Mei [35] studied the effect of lumped end mass on bending vibrations of a Timoshenko beam using the wave approach. Furthermore, Mei [36] used the wave method for calculating the natural frequencies and mode shapes of single-story multi-bay planar frame structures. Recently, Mei [37] studied the vibration of single-story multi-bay planar frame structures via the wave approach, in which the rotary inertia, and shear deformation effects were taken into account. A modified wave approach was presented by Bahrami et al. [38] to analysis a non-uniform Euler beam. Also, the wave propagation technique was used for natural frequency calculation of thin cylindrical shells [39]. This method was also applied for coupled vibration of fluid-filled shells [40], submerged shells [41] and cross-ply laminated composite shells [42]. Xuebin [43] analyzed the free vibration of a circular cylindrical shell via the wave propagation method based on Flugge theory. Recently, Bahrami et al. [44] used the wave method for free vibration of circular and annular membranes to obtain the natural frequencies of these structures. In another work, Bahrami and Teimourian [45] showed that the wave method can be extended to non-uniform circular and annular membranes.

It is noticeable to mention that the wave propagation phenomenon which is applied by so many researchers in the literature for CNTs [46–53], is very different from the wave propagation method. The wave propagation method uses propagation and reflection matrices to analyze the behavior of the bodies while the wave propagation analysis is based on the investigation of wave behavior in the structures by investigating the dispersion equation.

According to present literature reviews and to the best of the authors' knowledge, there are only two papers based on wave propagation method for vibration and wave reflection analysis of nanostructures. Recently, Ilkhani et al. [54] studied the effect of small scale on the vibration and wave reflection in thin rectangular nanoplate. Also, Bahrami and Teimourian [55] investigated the nonlocal scale effect on buckling, vibration and wave reflection in Euler–Bernoulli nanobeams. In reality, most of the carbon nanotubes have thick thickness and Euler–Bernoulli beam theory cannot predict the actual behavior of such nanostructures. As a result, the power reflection results of these two papers [54,55] cannot be applied to a thick nanostructure and there is still no literature about energy transmission and reflection in thick nanostructures. In this paper, the wave propagation technique is applied to a thick nanoscale structure in order to obtain the propagation and reflection matrices. The obtained propagation and reflection matrices will be helpful for future work correlated with wave power transmissions and reflections in nanoscale structure. Moreover, the natural frequencies obtained by this method are compared with the results in the literature. Finally, the small scale effect on the reflected power of an incident wave upon different boundary conditions is studied in details.

2. Mathematical formulations

2.1. Equation of motion

According to the nonlocal theory proposed by Eringen [5], the stress at a reference point x in an elastic medium depends on the strain at every point of the body. Based on this theory, the stress at a point is defined as:

$$\sigma_{ij} = \int \lambda(|x - x'|, a) C_{ijkl} \varepsilon_{kl}(x') dV(x') \tag{1}$$

where σ_{ij} and ε_{kl} are stress and strain tensors, C_{ijkl} is fourth order elastic modulus tensor, $\lambda(|x - x'|, a)$ is nonlocal kernel which weights the classical strains with x around the volume V and a is the material constant. The material constant is defined as $e_0 a/l$ where a is internal characteristic length which depends on lattice parameter, granular size and C–C bonds, l is external characteristic length and e_0 is a constant correlated by material type. The parameter $e_0 a$ is known as small scale. A certain value for small scale is not available and for any type of analysis this will be found by comparing the results of continuum modeling with atomistic ones. The nonlocal kernel function was defined by Eringen [5]:

$$\lambda(|x|, a) = \left(2\pi l^2 a^2\right)^{-1} K_0\left(\frac{\sqrt{x \cdot x}}{la}\right) \tag{2}$$

where K_0 is modified Bessel function. By integrating Eq. (1) using Eq. (2), the constitutive equation can be obtained as:

$$\left(1 - (e_0 a)^2 \nabla^2\right) \boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} \tag{3}$$

where ∇^2 is Laplacian operator. The displacement fields of the elementary Timoshenko beams can be considered as: