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Soft and hard anisotropic interface in composite materials

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ABSTRACT

For a large class of composites, the adhesion at the fiber–matrix interface is imperfect i.e. the continuity conditions for displacements and often for stresses is not satisfied. In the present contribution, effective elastic moduli for this kind of composites are obtained by means of the Asymptotic Homogenization Method (AHM). Interaction between fiber and matrix is considered for linear elastic fibrous composites with parallelogram periodic cell. In this case, the contrast or jump in the displacements on the boundary of each phase is proportional to the corresponding component of the tension on the interface. A general anisotropic behavior of the interphase is assumed and the interface stiffnesses are explicitly given in terms of the elastic constants of the interphase. The constituents of the composites exhibit transversely isotropic properties. A doubly periodic parallelogram array of cylindrical inclusions is considered. Comparisons with theoretical and experimental results verified that the present model is efficient for the analysis of composites with presence of imperfect interface and parallelogram cell. The present method can provide benchmark results for other numerical and approximate methods.

1. Introduction

In this work, micromechanical analysis method is applied to a periodic composite with unidirectional fibers and parallelogram cells. The analytical expressions of the homogenized elastic properties are calculated for two phase composite with hard and soft interfaces. The Asymptotic Homogenization Method (AHM), for two-phase fibrous periodic composites with imperfect adhesion and oblique cell is used for the calculation of the plane elastic effective coefficients. This contribution is an extension of previous works by the authors (Rodriguez-Ramos et al., 2011 [1], Guinovart-Diaz et al., 2011 [2]), where only the perfect contact was considered for the antiplane problem. Besides, the present investigation is different of those of Lopez-Realpozo et al., 2011 [3] and Rodriguez-Ramos et al., 2013 [4] since the plane problem is solved for the calculation of the effective coefficients for composites with parallelogram cell. The novelty of the present work is that the imperfection of the interface in the composite with parallelogram cell is taken into account introducing two spring-type stiffnesses (K_n,K_t) for plane problems.

Using a classical approach [5], the spring parameters can be identified from a three phase problem where the interphase coating the fiber is very thin. The paper is organized as follows. In the first part of the paper the derivation of the contact law mechanically equivalent to the interphase coating the fiber is reviewed on the basis of an energy method [6]. The method allows obtaining the spring-type interface law for a general anisotropic behavior of the interphase and the interface parameters (K_n,K_t) are explicitly given in terms of the elastic constants of the interphase.

The results of the micromechanical analysis presented in the second part of this paper are mainly focused on the impact of the

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arrangement of the fibers and the mechanic imperfection at the interface on the plane properties in the composites. Moreover, the theoretical approach is validated with some theoretical models.

2. Modeling of imperfect contact

The interphase coating the fiber is represented as a thin layer B^{ε} with uniform small thickness $\varepsilon \ll 1$ and cross-section A. The interphase joins the fiber and the matrix, assumed to occupy the reference configurations S_1^{ε} and S_2^{ε}, respectively.

Let Γ_1^{ε}, Γ_2^{ε} be the interfaces between the adhesive and the adherents and let $S^{\varepsilon} = S_1^{\varepsilon} \cup S_2^{\varepsilon} \cup B^{\varepsilon} \cup \Gamma_1^{\varepsilon} \cup \Gamma_2^{\varepsilon}$ denote the composite made of the adhesive and the two adherents.

Adhesive and adherents are assumed to be perfectly bonded in order to ensure the continuity of the displacement and stress vector fields across Γ_1^{ε}, Γ_2^{ε}.

Let (O, i_1, i_2, i_3) be an orthonormal Cartesian basis and let (O, x_1, x_2, x_3) be the coordinates a particle. The origin is taken at the center of the interphase midplane and the x_3- axis is perpendicular to the interphase midplane.

The materials are homogeneous and linear elastic with C^1_{ijkl}, C^2_{ijkl} and C^{ε}_{ijkl} the elasticity tensors of the adherents and of the interphase, respectively. The elasticity tensors are assumed to be symmetric, with the minor and major symmetries, and positive definite.

The adhesive is assumed to be *soft*, i.e. $C^{\varepsilon}_{ijkl} = \varepsilon C_{ijkl}$ with C_{ijkl} independent of ε.

The adherents are subjected to a body force density $f: S_1^{\varepsilon} \cup S_2^{\varepsilon} \to \mathbb{R}^3$ and to a surface force density $g: \Gamma_{\varepsilon} \to \mathbb{R}^3$ on $\Gamma_g^{\varepsilon} \subset (\partial S_1^{\varepsilon} \Gamma_1^{\varepsilon}) \cup (\partial S_2^{\varepsilon} \Gamma_2^{\varepsilon})$. Body forces are negligible in the adhesive.

On the complementary part $\Gamma_u^{\varepsilon} \subset (\partial S_1^{\varepsilon} \Gamma_1^{\varepsilon}) \cup (\partial S_2^{\varepsilon} \Gamma_2^{\varepsilon}) \Gamma_g$ homogeneous boundary conditions are assigned: $u^{\varepsilon} = 0$ on Γ_u^{ε}, where $u^{\varepsilon}: S^{\varepsilon} \to \mathbb{R}^3$ is the displacement field defined from S^{ε}. The sets Γ_g^{ε}, Γ_u^{ε} are assumed to be located far from the interphase and the fields of the external forces are sufficient regularity to ensure the existence of equilibrium configuration.

The approach used in Ref. [6] to obtain the contact law is based on the fact that stable equilibrium configurations of the composite assemblage minimize the total energy:

$$E^{\varepsilon}(u) = \int_{S_1^{\varepsilon}} \left(\tfrac{1}{2} C^1_{ijkl} u_{i,j} u_{k,l} - f_i u_i \right) dV_x + \int_{S_2^{\varepsilon}} \left(\tfrac{1}{2} C^2_{ijkl} u_{i,j} u_{k,l} - f_i u_i \right) dV_x - \int_{\Gamma_g^{\varepsilon}} g_i u_i dA_x + \int_{B^{\varepsilon}} \left(\tfrac{1}{2} \varepsilon C_{ijkl} u_{i,j} u_{k,l} \right) dV_x,$$

in the space of kinematically admissible displacements:

$$V^{\varepsilon} = \left\{ u \in H\left(S^{\varepsilon}; \mathbb{R}^3\right) : u = 0 \text{ on } \Gamma_u^{\varepsilon} \right\},$$

where $H(S^{\varepsilon}; \mathbb{R}^3)$ is the space of the vector-valued functions on the set S^{ε} which are continuous and differentiable as many times as necessary. Under suitable regularity assumptions, the existence of a unique minimizer $u \in V^{\varepsilon}$ is ensured [7].

For the asymptotic analysis, it is convenient to introduce the following change of variables $\hat{p}: (x_1, x_2, x_3) \to (z_1, z_2, z_3)$ in the adhesive:

$$z_1 = x_1, \; z_2 = x_2, \; z_3 = x_3/\varepsilon,$$

which gives

$$\frac{\partial}{\partial z_1} = \frac{\partial}{\partial x_1}, \; \frac{\partial}{\partial z_2} = \frac{\partial}{\partial x_2}, \; \frac{\partial}{\partial z_3} = \varepsilon \frac{\partial}{\partial x_3}.$$

A change of variable $\bar{p}: (x_1, x_2, x_3) \to (z_1, z_2, z_3)$ is also introduced in the adherents:

$$z_1 = x_1, \; z_2 = x_2, \; z_3 = x_3 \pm 1/2(1-\varepsilon),$$

where the plus (minus) sign applies whenever $x \in S_1^{\varepsilon}$ $(x \in S_2^{\varepsilon})$ and one has

$$\frac{\partial}{\partial z_1} = \frac{\partial}{\partial x_1}, \; \frac{\partial}{\partial z_2} = \frac{\partial}{\partial x_2}, \; \frac{\partial}{\partial z_3} = \frac{\partial}{\partial x_3}.$$

After the change of variables $\hat{p}$, the interphase occupies the domain

$$B = \left\{ (z_1, z_2, z_3) \in \mathbb{R}^3 : (z_1, z_2) \in A, |z| < 1/2 \right\},$$

and the adherents occupy the domains $S_{1,2} = S^{\varepsilon}_{1,2} \pm 1/2(1-\varepsilon) i_3$. The sets $\Gamma_{1,2} = \{(z_1, z_2, z_3) \in \mathbb{R}^3 : (z_1, z_2) \in A, z_3 = \pm 1/2\}$ are taken to denote the interfaces between B, and S_{1,2}, and $S = S_1 \cup S_2 \cup B \cup \Gamma_1 \cup \Gamma_2$ is called the rescaled configuration of the composite body. Lastly, Γ_u and Γ_{ε} indicate the images of Γ_u^{ε} and Γ_g^{ε} under the change of variables, $\bar{f} := f \circ \bar{p}^{-1}$ and $\bar{g} = g \circ \bar{p}^{-1}$ the rescaled external forces.

Using the changes of variables given by $\bar{p}$, $\hat{p}$ and denoting $\bar{u} = u^{\varepsilon} \circ \bar{p}^{-1}$ and $\hat{u} = u^{\varepsilon} \circ \hat{p}^{-1}$ the displacement fields from the rescaled adhesive and adherents, respectively, the total energy takes the rescaled form:

$$E^{\varepsilon}(\hat{u}^{\varepsilon}, \bar{u}^{\varepsilon}) = \int_{S_1} \left(\tfrac{1}{2} C^1_{ijkl} \bar{u}^{\varepsilon}_{i,j} \bar{u}^{\varepsilon}_{k,j,l} - \bar{f}_i \bar{u}^{\varepsilon}_{i,j} \right) dV_z + \int_{S_2} \left(\tfrac{1}{2} C^2_{ijkl} \bar{u}^{\varepsilon}_{i,j} u^{\varepsilon}_{k,j,l} - \bar{f}_i \bar{u}^{\varepsilon}_{i,j} \right) dV_z - \int_{\Gamma_{\varepsilon}} \bar{g}_i \bar{u}^{\varepsilon}_i dA_z + \int_{B^{\varepsilon}} \tfrac{1}{2} \left(\varepsilon^{-1} K^{33}_{ki} \hat{u}^{\varepsilon}_{k,3} \hat{u}^{\varepsilon}_{i,3} + 2 K^{\alpha 3}_{ki} \hat{u}^{\varepsilon}_{i,3} \hat{u}^{\varepsilon}_{k,\alpha} + \varepsilon K^{\alpha\beta 3}_{ki} \hat{u}^{\varepsilon}_{i,\alpha} \hat{u}^{\varepsilon}_{k,\beta} \right) dV_z,$$