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Coupled longitudinal-transverse-rotational behaviour of shear deformable microbeams

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ABSTRACT

In this paper, for the first time, the nonlinear motion characteristics of a hinged-hinged third-order shear deformable microbeam are examined, based on the modified couple stress theory and the third-order shear deformation theory. The extensibility of the microbeam is modelled by taking into account the longitudinal displacement. The nonlinear equations governing the longitudinal, transverse, and rotational motions are derived by means of Hamilton's principle in conjunction with the modified couple stress theory (to take into account small-scale effects). The three coupled nonlinear partial differential equations are discretized via the Galerkin method and the resulting set of ordinary differential equations is solved by means of the pseudo-arclength continuation technique and via direct time-integration. The effects of the system parameters on the behaviour of the microbeam are studied. Results are presented in the form of frequency-responses and force-responses. Points of interest in the parameter space are also highlighted in the form of time histories, phase-plane portraits, and fast Fourier transforms (FFTs). Moreover, the similarities and differences in the response of the system obtained via the modified couple stress and classical continuum mechanics theories are discussed.

1. Introduction

The demand for microscale continuous elements [1,2] such as microbeams [3,4], microplates [5–11], and microshells [12] is increasing rapidly, mainly due to their growing application in industry. In particular, microbeams are present in biosensors, microactuators, microswitches, vibration and shock sensors, and biomechanical devices. In almost all of these applications, the microscale structure is subject to a source of energy, causing these elements to either deform or oscillate; analyzing the motion/deformation behaviour of these systems is hence essential for optimization purposes and also to achieve better design factors.

One important barrier in the theoretical analysis of microscale continuous elements is their inherent dynamical dependence on size; many experiments [13,14] reported that these systems display a size-dependent deformation behaviour, which cannot be predicted by the classical continuum mechanics theories – the modified couple stress theory is employed in this paper to overcome this problem by taking into account small-scale effects.

In some applications, the microscale continuous elements are subject to forces from other non-ideal elements or supports; these effects are taken into account in this study by additionally supporting the microbeam by an intermediate nonlinear spring.

The motion characteristics of microbeams have been investigated extensively in the literature. These studies can mainly be classified into two groups in terms of the models being considered. The first class analyzed the motion characteristics of microbeams based on either the Euler–Bernoulli or Timoshenko beam models [15–20]. The second class, on the other hand, employed higher-order shear deformation beam theories in the modelling.

1.1. Literature review on the first class

The number of studies belonging to the first class of analysis is quite large [21–26]. Reviewing the linear aspects of the problem, for example, Kong et al. [27] obtained the natural frequencies of a Euler–Bernoulli microbeam based on the modified couple stress theory. A similar analysis was performed by Ma et al. [28], but for a Timoshenko microbeam. Ke and Wang [29] examined the dynamic stability of a functionally graded Timoshenko microbeam employing the modified couple stress theory. Wang et al. [30] developed a

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Timoshenko microbeam model based on a strain gradient elasticity theory and examined its static bending and free oscillations. These studies were extended to nonlinear models, for instance, by Ramezani [31], who utilized the method of multiple timescales to examine the nonlinear free dynamics of a Timoshenko microbeam on the basis of a strain gradient elasticity theory. Asghari et al. [32] contributed to the field by studying the size-dependent nonlinear dynamics of a Timoshenko microbeam based on the modified couple stress theory. Mohammadi and Mahzoon [33] investigated the thermal effects on the nonlinear post-buckling behaviour of a Euler–Bernoulli microbeam based on the modified strain gradient theory.

1.2. Literature review on the second class

The second class of analysis, concerned with the analysis of higher-order microbeam models, is not large. For example, Nateghi et al. [34] investigated the size-dependent buckling behaviour of functionally graded microbeams employing the classical theory and first- and third-order shear deformation beam theories. Salamat-talab et al. [35] examined the linear static and dynamic responses of a shear deformable functionally graded microbeam employing the modified couple stress theory along with the third-order shear deformation theory. Şimşek and Reddy [36] analyzed the linear buckling response of a functionally graded microbeam embedded in an elastic medium based on the modified couple stress theory and a unified higher order beam theory. The investigations were continued by Sahmani and Ansari [37], who investigated the linear buckling response of a third-order shear deformable functionally graded microbeam subject to temperature variations, employing a strain gradient elasticity theory. Mohammad-Abadi and Daneshmehr [38] employed the modified couple stress theory in order to analyze the size-dependent buckling behaviour of higher-order microbeams. Akgöz and Civalek [39] contributed to the field by analyzing the buckling behaviour of a higher-order shear deformable functionally graded microbeam utilizing new shear correction factors. Zhang et al. [40] developed a size-dependent functionally graded microbeam model based on a strain gradient elasticity theory and a third-order shear deformation theory.

1.3. Contributions of the current study to the field

This paper, for the first time, examines the nonlinear motion characteristics of an extensible third-order shear deformable microbeam with an intermediate *spring-support*. Based on the modified couple stress theory, the microbeam is modelled by means of the third-order shear deformation theory retaining the longitudinal displacement and inertia; this is also the first time that *extensibility* is retained in the nonlinear analysis of third-order shear deformable microbeams. Taking into account small-size effects, via the modified couple stress theory, the potential and kinetic energies as well as works due to damping and external excitation are obtained in terms of the longitudinal and transverse displacements and rotation. Hamilton's principle is then employed to derive the longitudinal, transverse, and rotational equations of motion. These three coupled nonlinear partial differential equations are then discretized through use of the Galerkin method, yielding a set of second-order nonlinear ordinary differential equations with coupled terms. This set is then, after diagonalizing the mass matrix, double-dimensionalized via a change of variables; the resultant equations are then solved via the pseudo-arclength continuation technique and a direct time-integration method, based on the variable step-size Runge-Kutta scheme. Numerical results are presented in the form of frequency-responses, force-responses, time traces, phase-plane portraits, and fast Fourier transforms (FFTs). The effect of the linear and nonlinear stiffness coefficients of the spring-support as well as its location on the motion characteristics of the shear deformable microbeam is examined. Moreover, a comparison is made between the motion characteristics of the microbeam based on the modified couple stress and classical continuum mechanics theories.

2. Nonlinear coupled equations of motion

Fig. 1 shows a shear deformable microbeam of length L, thickness h, Young's modulus E, cross-sectional area A, and area moment of inertial I. The microbeam is simply supported at both ends and subjected to a distributed harmonic excitation load per unit length $F(x)\cos(\omega t)$, in the transverse direction. A nonlinear spring is attached to the centreline of the microbeam at a distance x_s from the left end; k_1 and k_2 are the linear and nonlinear stiffness coefficients of the spring-support. $u(x,t)$ and $w(x, t)$ represent the displacements in longitudinal and transverse directions and $\phi(x, t)$ denotes the rotation of the transverse normal.

The equations of motion have been derived assuming that: (1) the cross-sectional area is uniform along the length of the microbeam [41]; (2) a geometric nonlinearity, due to the mid-plane stretching is considered [42–47]; (3) a third-order shear deformation model is considered; (4) there is no warping in the system [48–50]; (5) the spring is attached to the centreline of the microbeam with the effect only in the transverse direction.

The components of the displacement vector $\mathbf{u}$ of a point located at a distance z from the mid-plane are given by

$$\begin{aligned} u_1(x,z,t) &= u(x,t) + z\phi(x,t) - \frac{4}{3h^2}z^3\left(\phi(x,t) + \frac{\partial w(x,t)}{\partial x}\right), \\ u_2(x,z,t) &= 0, \\ u_3(x,z,t) &= w(x,t), \end{aligned} \tag{1}$$

which results in the following non-zero components of the strain tensor

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + z\frac{\partial \phi}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right), \\ \varepsilon_{xz} &= \varepsilon_{zx} = \frac{1}{2}\left(1 - \frac{4z^2}{h^2}\right)\left(\phi + \frac{\partial w}{\partial x}\right). \end{aligned} \tag{2}$$

The symmetric curvature tensor $\boldsymbol{\chi}$ can be written as a function of the displacement vector as [51]

$$\boldsymbol{\chi} = \frac{1}{4}\left([\nabla(\mathrm{curl}(\mathbf{u}))] + [\nabla(\mathrm{curl}(\mathbf{u}))]^{\mathrm{T}}\right), \tag{3}$$

which according to Eq. (1), gives the following non-zero components of the symmetric curvature tensor

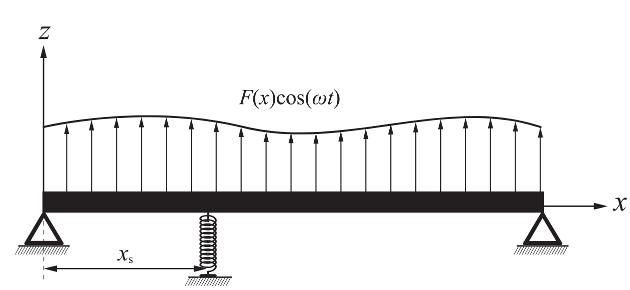

Fig. 1. Schematic representation of a shear deformable microbeam, additionally constrained by a nonlinear spring-support.