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The influence of the normal strain effect on the control and design optimization of functionally graded plates

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ABSTRACT

A multiobjective design and control optimization problem for functionally graded (FG) plates is presented using a first-order plate theory including the normal strain effect. The aim of the optimization is to minimize the vibrational response and to maximize the buckling loads of FG plates with constraints on the control energy and plate thickness. An integrated approach for the simultaneous design and active control optimization is presented to determine the optimal level of a closed loop control function. Plate thickness and a homogeneity parameter of FG plates are used as design variables. Numerical results for the optimal control force and the total energy of FG plates are presented in various cases of boundary conditions. The influence of the normal strain effect on the accuracy of the obtained results is illustrated. The effectiveness of the present control and design procedures are examined.

1. Introduction

Functionally graded materials (FGMs) [1] are made of advanced composites in which the material properties and thus the mechanical properties vary smoothly and continuously from one surface to the other. These special characteristics make them preferable to conventional composite materials. In recent years, functionally graded materials (FGMs) have gained considerable attention in many engineering applications, and are considered as a potential structural material for future high-speed spacecraft and power generation industries. Several studies have been performed to analyze the behavior of functionally graded plates [2–6].

The rapid development in various industrial fields requires new materials that can serve useful functions under certain conditions. In the aerospace industry and many other engineering applications, the suppression of excessive vibrations occurring in large structures represents one of the most pressing and difficult problems facing structural designers. An effective means of suppressing excessive vibrations is by active structural control. Thus, there is need for new light materials possessing a high degree of flexibility and with very low natural damping. These factors motivated the development for more accurate tools of analysis and rigorous design methods [7–12].

The strong interaction between the optimal design and the active structural control is considered the most effective means for improving the performance of composite structures. Thus, a number of integrated approaches for the simultaneous design and control optimization have been manifested in literature. Grandhi [13] studied the structural and control optimization of space structures. Sloss et al. [14] presented an integrated approach to solve multiobjective design and control optimization for many composite structures. Adali et al. [15,16], Sloss et al. [17], Sadek et al. [18], and Fares et al. [19–21] have studied the optimal design and control optimization of composite laminated plates and shells subjected to thermo mechanical loadings. In another work of Fares et al. [22], a nonlinear optimization scheme is presented to solve multiobjective design and control problems for composite laminated plates. Other studies on this topic may be found in Refs. [23–26].

Many studies indicate that the inclusion of the normal strain and the transverse shear effects are very important for predicting accurate buckling and vibrational responses of composite plates and shells, particularly, for those made of FGMs, see e.g., the reference [27]. The work [27] showed that neglecting these effects may cause errors in predicting the deflections and frequencies in FG plates reaching to 30%. In addition, the different boundary conditions contribute significantly in occurrence of the out-of-plane deflections. However, most studies related to the topic of design and control optimization of composite plates and shells, were carried out for special cases of boundary conditions, and are based on classical theories which neglect the shear deformation and normal strain effects.

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The current work deals with the optimal design and control of FG orthotropic rectangular plates in various cases of boundary conditions. The present formulation is based on a consistent first-order plate theory including the shear deformation and normal strain effects [27]. The design and control objectives are to maximize the buckling load and to minimize the dynamic response with minimum expenditure of control energy. The total elastic energy is taken as a measure of the dynamic response. A quadratic functional of the total energy is specified as a control performance index. The expenditure of the control energy is limited by attaching to the control objective a functional including the control force. The necessary and sufficient condition for the optimal stabilization in the Liapunov–Bellman sense [28] is used to determine the optimal control force, controlled deflections and corresponding elastic energy. The design procedure aims to minimize the controlled elastic energy and simultaneously, maximize the buckling loads using a design function including the two responses. This design optimization may be suitable for some large space structures requiring that the plate must resist buckling due to high in-plane forces, and in the same time, it can suppress its excessive vibration. Numerical examples are given to assess the efficiency of the present design and control approach for FG plates with various boundary conditions.

1.1. Formulation and basic equations

Consider an elastic plate of uniform thickness h, length a and width b. The plate is composed of an orthotropic elastic material varying across the thickness direction. Let a Cartesian coordinates system (x, y, z) are taken such that the mid-plane coincides with the xy plane and the plate occupies the following region:

$$0 \le x \le a, \quad 0 \le y \le b, \quad -h/2 \le z \le h/2.$$

Let the upper surface of the plate ($z = h/2$) be subjected to a transverse distributed load $q(x,y,z)$ which may be taken as a control force. Also, there are compressive forces P_1 and P_2 (per unit length) acting on the edges of the mid-plane.

The present formulation is based on a first-order theory including a normal strain and shear deformation effects derived using a mixed variational approach. This theory preserves the transverse shear stresses vanish on the top and the bottom of the plate surfaces. Therefore, this theory does not require introducing any shear correction factor into the formulation. The displacement field is taken in the form [27]:

$$u = u_0 + \psi z, \quad v = v_0 + \varphi z, \quad w = w_0 + \alpha w_1 z + \beta w_2 z^2. \tag{1}$$

where (u, v, w) are the displacements along x, y and z directions respectively, (u_0, v_0, w_0) are the displacements of a point on the mid-plane, and (ψ, φ) are the slopes in the xz and yz planes due to bending only (slope rotations). The constants α and β may take the values 0 or 1 to study the effect of the normal strain on the control process.

The infinitesimal strains ε_{ij} associated with displacements (1) are given by:

$$\begin{aligned}
&\varepsilon_{11} = \frac{\partial u_0}{\partial x} + \frac{\partial \psi}{\partial x} z, \quad \varepsilon_{22} = \frac{\partial v_0}{\partial y} + \frac{\partial \varphi}{\partial y} z, \quad \varepsilon_{33} = \alpha w_1 + 2\beta w_2 z,\\
&\varepsilon_{12} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) + \left(\frac{\partial \psi}{\partial y} + \frac{\partial \varphi}{\partial x}\right) z,\\
&\varepsilon_{13} = \psi + \frac{\partial w_0}{\partial x} + z\alpha \frac{\partial w_1}{\partial x} + z^2 \beta \frac{\partial w_2}{\partial x},\\
&\varepsilon_{23} = \varphi + \frac{\partial w_0}{\partial y} + z\alpha \frac{\partial w_1}{\partial y} + z^2 \beta \frac{\partial w_2}{\partial y}.
\end{aligned} \tag{2}$$

Moreover, the stresses are taken in the following form:

$$\begin{aligned}
&\sigma_{ij} = \frac{N_{ij}}{h} + \frac{12 M_{ij}}{h^3} z, \quad \sigma_{i3} = \frac{3 Q_{i3}}{2h}\left[1 - \left(\frac{z}{h/2}\right)^2\right], \quad (i,j = 1,2)\\
&\sigma_{33} = \left(\frac{3N_{33}}{2h} + \frac{30 M_{33}}{h^3} z\right)\left[1 - \left(\frac{z}{h/2}\right)^2\right] + \frac{q_1}{4}\left[\left(\frac{z}{h/2}\right)^2 - \left(\frac{z}{h/2}\right) - 2\right]\\
&\quad \times \left(1 + \frac{z}{h/2}\right) + \frac{q_2}{4}\left[\left(\frac{z}{h/2}\right)^2 + \left(\frac{z}{h/2}\right) - 2\right]\left(1 - \frac{z}{h/2}\right).
\end{aligned} \tag{3}$$

The governing equations of this system are derived using a dynamic version of the mixed variational principle of Ressiner [27]:

$$\begin{aligned}
&\delta u_0 : \frac{\partial N_{11}}{\partial x} + \frac{\partial N_{12}}{\partial y} = I_1 \ddot{u}_0 + I_2 \ddot{\psi},\\
&\delta v_0 : \frac{\partial N_{12}}{\partial x} + \frac{\partial N_{22}}{\partial y} = I_1 \ddot{v}_0 + I_2 \ddot{\varphi},\\
&\delta w_0 : \frac{\partial Q_{13}}{\partial x} + \frac{\partial Q_{23}}{\partial y} + q = I_1 \ddot{w}_0 + \alpha I_2 \ddot{w}_1\\
&\qquad + \beta I_3 \ddot{w}_2 - \left(P_1 \frac{\partial^2 w_0}{\partial x^2} + P_2 \frac{\partial^2 w_0}{\partial y^2}\right),\\
&\delta \psi : \frac{\partial M_{11}}{\partial x} + \frac{\partial M_{12}}{\partial y} - Q_{13} = I_2 \ddot{u}_0 + I_3 \ddot{\psi},\\
&\delta \phi : \frac{\partial M_{12}}{\partial x} + \frac{\partial M_{22}}{\partial y} - Q_{23} = I_2 \ddot{v}_0 + I_3 \ddot{\phi},\\
&\delta w_1 : hq - N_{33} = I_2 \ddot{w}_0 + \alpha I_3 \ddot{w}_1 + \beta I_4 \ddot{w}_2,\\
&\delta w_2 : \frac{h^2}{20}\left(\frac{\partial Q_{13}}{\partial x} + \frac{\partial Q_{23}}{\partial y}\right) - 2M_{33} + \frac{9h^2}{20} q\\
&\qquad = I_3 \ddot{w}_0 + \alpha I_4 \ddot{w}_1 + \beta I_5 \ddot{w}_2.
\end{aligned} \tag{4}$$

Moreover, the constitutive equation equations are:

$$\begin{aligned}
&\begin{bmatrix} [N] \\ [M] \end{bmatrix} = \begin{bmatrix} [A_{ij}] & [B_{ij}] \\ [B_{ij}] & [D_{ij}] \end{bmatrix}^{-1} \begin{bmatrix} [R_1] \\ [R_2] \end{bmatrix},\\
&[A_{ij}] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 \\ A_{12} & A_{22} & A_{23} & 0 \\ A_{13} & A_{23} & A_{33} & 0 \\ 0 & 0 & 0 & A_{66} \end{bmatrix}, \quad [N] = \begin{bmatrix} N_{11} \\ N_{22} \\ N_{33} \\ N_{12} \end{bmatrix}^T\\
&[B_{ij}] = \begin{bmatrix} B_{11} & B_{12} & B_{13} & 0 \\ B_{12} & B_{22} & B_{23} & 0 \\ B_{13} & B_{23} & B_{33} & 0 \\ 0 & 0 & 0 & B_{66} \end{bmatrix}, \quad [M] = \begin{bmatrix} N_{11} \\ N_{22} \\ N_{33} \\ N_{12} \end{bmatrix}^T \qquad (5a)\\
&[D_{ij}] = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 \\ D_{12} & D_{22} & D_{23} & 0 \\ D_{13} & D_{23} & D_{33} & 0 \\ 0 & 0 & 0 & D_{66} \end{bmatrix},\\
&[R_1] = \left[\tfrac{\partial u_0}{\partial x} - F_{13} \quad \tfrac{\partial v_0}{\partial y} - F_{23} \quad w_1 \alpha - F_{33} \quad \tfrac{\partial u_0}{\partial y} + \tfrac{\partial v_0}{\partial x}\right]^T,\\
&[R_2] = \left[\tfrac{\partial \psi}{\partial x} - K_{13} \quad \tfrac{\partial \phi}{\partial y} - K_{23} \quad 2w_2 \beta - K_{33} \quad \tfrac{\partial \psi}{\partial y} + \tfrac{\partial \phi}{\partial x}\right]^T,
\end{aligned}$$