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An engineering formula for the stress concentration factor of orthotropic composite plates

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ABSTRACT

An engineering formula for the theoretical stress concentration factor of orthotropic notched plates under tension is provided, as a function of the material elastic constants and the K_t of the corresponding isotropic case. The accuracy and limits of applicability of the new solution are discussed by comparison to data from the literature and results from FE analyses on notched geometries of practical interests. The proposed solution represents a very useful tool to estimate the stress concentration factor of notched orthotropic plates, composite orthotropic laminae, orthotropic unidirectional laminates and homogenised orthotropic composite laminates.

1. Introduction

Geometrical variations, such as notches, grooves or holes, unavoidably exist in engineering components, being often responsible for crack formation under static and cyclic conditions.

This is the reason why the study of stress distributions around notches has received a large attention in the past and recent literature. Blunt cracks in isotropic plates have been analysed by Creager and Paris [1] who were able to give the closed form solution for the local stress fields for Mode I, II and III loadings. The above mentioned solution was later extended to blunt V-notches, including the effect of the notch opening angle, both under linear elastic [2–5] and elastic plastic conditions [6,7]. It is also worth mentioning that a large number of solutions for central holes in isotropic plates is present in the book by Savin [8].

Moving to orthotropic plates, comparatively few works can be found in the literature, mainly oriented to refine the classical analysis for anisotropic plates with elliptical holes by Lekhnitskii [9] and to make it applicable to composite laminates (see, among the others, Bonora et al., [10,11], Chern and Tuttle [12]). More recently, Ukadgaonker and Rao [13] carried out an analytical study of the stress distributions in an orthotropic plate with triangular holes, while the case of an irregular shaped hole has been later considered by Ukadgaonker and Rao [14] and by Ukadgaonker and Kakhandki [15], where an excellent literature review on the topic can be found, as well.

The design against fatigue or brittle failure of blunt notched engineering components is often based on strength criteria, according to which the stress values in the stress concentration regions are compared to the fatigue or static strength of the base material, respectively. This process is complicated by the fact that the stress state close to a notch is inherently multiaxial and, under such a stress state, the fatigue behaviour of composite materials might be very complex [16–21].

In the engineering practice, the maximum stress at a blunt notch root is correlated to the nominal stress using the theoretical stress concentration factor. However, different from isotropic materials, in orthotropic plates K_t depends not only on the geometry but also on the elastic material properties [8,9,22–24]. This hampers the possibility for engineers to use design charts or approximated relationships, as those provided in Refs. [25–30], allowing a rapid evaluation of K_t.

Although advanced computational technology has made it possible to calculate the stress concentration factor for any notched geometry and material, practical expressions, providing a rapid evaluation of this parameter, remain very useful in the engineering practice. To this end, starting from some analytical derivations based on classical solutions of the orthotropic theory of elasticity [9], an engineering formula to estimate the theoretical stress concentration factor of orthotropic notched plates under tension is provided, involving the material elastic properties and the K_t of the corresponding isotropic case, which is geometry-dependent, only.

Accuracy and limits of applicability are discussed by comparing the approximate solution to numerical results from the literature

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and from FE analyses carried out by the authors, considering notched plates of practical interest.

The proposed solution represents a useful tool to estimate the stress concentration factor of notched orthotropic plates, composite orthotropic laminae, orthotropic unidirectional laminates and homogenised orthotropic composite laminates.

2. Material behaviour

Although the notch tip stress state in a thick plate is always, by very nature, three-dimensional, the adoption of two dimensional hypotheses, such as plane stress or plane strain, allows to remove many difficulties encountered in the three-dimensional anisotropic elasticity theory. Moreover under particular conditions, plane hypotheses can be representative of the actual three-dimensional behaviour [31,32]. Accordingly, in the present work notched orthotropic plates under plane stress or plane strain conditions are considered only.

For plane stress problems ($\sigma_3 = \tau_{13} = \tau_{23} = 0$) the elastic orthotropic stress–strain relationships can be formulated on the basis of four independent elastic constants:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{1}$$

For plane strain problems ($\varepsilon_3 = \gamma_{13} = \gamma_{23} = 0$), instead, Hooke's law in terms of compliance matrix would read as:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{2}$$

where the constants B_{ij} can be expressed in terms of the compliances S_{ij}:

$$\begin{aligned} B_{11} &= \frac{S_{11}S_{33} - S_{13}^2}{S_{33}} \qquad B_{12} = \frac{S_{12}S_{33} - S_{13}S_{23}}{S_{33}} \\ B_{22} &= \frac{S_{22}S_{33} - S_{23}^2}{S_{33}} \qquad B_{66} = S_{66} \end{aligned} \tag{3}$$

Accordingly, from the mathematical point of view, the plane stress and plane strain problems are identical except for the values of elastic constants entering into the reduced strain–stress relations.

3. Analytical remarks

3.1. The elliptical hole in an infinite plate under tension

Consider an elliptical hole in an infinite orthotropic plate under tension, the direction of the far applied uniaxial tension equating the 1 principal elasticity direction. The stress concentration factor for this problem, referred to the gross section, is [9]:

$$K_{tg} = 1 + (\beta_1 + \beta_2)\sqrt{\frac{a}{\rho}} \tag{4}$$

where a is the notch depth (major semi-axis of the ellipse), ρ is the notch root radius and $u_{1,3} = \pm i\beta_1$ and $u_{2,4} = \pm i\beta_2$ are the conjugate roots of the following equation [8,9]:

$$T_{11}\mu^4 + (2T_{12} + T_{66})\mu^2 + T_{22} = 0 \tag{5}$$

Eq. (5) is the characteristic equation linked to the governing equation of the plane orthotropic theory of elasticity to be satisfied by the Airy stress function. Since μ_i always occur in conjugate pairs, it is possible to arrange, without loss of generality, that β_1 and β_2 are real and positive [9].

In Eq. (5) T_{ij} equate the terms of the compliance matrix, S_{ij}, for plane stress. In this case, invoking the engineering elastic constants:

$$S_{11} = \frac{1}{E_1} \quad S_{22} = \frac{1}{E_2} \quad S_{12} = \frac{-\nu_{21}}{E_2} = -\frac{-\nu_{12}}{E_1} \quad S_{66} = \frac{1}{G_{12}} \tag{6}$$

it results:

$$\zeta = \beta_1 + \beta_2 = \sqrt{2\sqrt{\frac{E_1}{E_2}} - 2\nu_{12} + \frac{E_1}{G_{12}}} \tag{7}$$

For plane strain conditions instead, $T_{ij} = B_{ij}$. It is worth noting that for an isotropic material $\beta_1 = \beta_2 = 1$, so that $\zeta = 2$.

Invoking the theoretical stress concentration factor for the corresponding isotropic case [33,34]:

$$\widetilde{K}_{tg} = 1 + 2\sqrt{\frac{a}{\rho}} \tag{8}$$

Eq. (4) can be conveniently re-written in the following form:

$$K_{tg} = 1 + \frac{\zeta}{2}(\widetilde{K}_{tg} - 1) \tag{9}$$

One should note that in the case of a wide orthotropic composite plate with a central hole $\widetilde{K}_{tg} = 3$ and Eq. (9) gives:

$$K_t = 1 + \sqrt{2\sqrt{\frac{E_1}{E_2}} - 2\nu_{12} + \frac{E_1}{G_{12}}} \tag{10}$$

in agreement with the expression reported in Ref. [35] and widely used for composite materials.

3.2. The edge notch in a semi-infinite plate under tension

With reference to edge notches in semi-infinite orthotropic plates under tension Chiang [36] proved that the theoretical stress concentration factor can be written as:

$$K_{tg} = 1 + \zeta \int \frac{f'(\xi)}{\xi} d\xi \tag{11}$$

where f is the function describing the notch boundary. Accordingly, the function $f'(\xi)/\xi$ does not depend on the material elastic properties, so that Eq. (9) holds valid also for semi-infinite plates with shallow edge notches of any shape.

3.3. The deep hyperbolic notch

The stress concentration factor, referred to the net section, for a tensioned orthotropic plate weakened by two symmetric deep hyperbolic notches is [9]:

$$K_{tn} = \sqrt{\frac{h}{\rho}} \frac{\beta_1^2 - \beta_2^2}{\beta_1 ArcTan\left(\beta_1\sqrt{\frac{h}{\rho}}\right) - \beta_2 ArcTan\left(\beta_2\sqrt{\frac{h}{\rho}}\right)} \tag{12}$$

where 2 h is the cross-sectional width and ρ is the notch root radius.

When the notch is sharp, ($\rho/h \to 0$), $ArcTan\left(\beta_i\sqrt{\frac{h}{\rho}}\right)$ tends towards $\pi/2$, and Eq. (12) can be approximated by:

$$K_{tn} \cong \frac{2}{\pi}\sqrt{\frac{h}{\rho}}(\beta_1 + \beta_2) \tag{13}$$

Under the same conditions, the stress concentration factor for the corresponding isotropic case is [34]:

$$\widetilde{K}_{tn} = \frac{4}{\pi}\sqrt{\frac{h}{\rho}} \tag{14}$$