Composites: Part B 68 (2015) 113–120

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Fracture mechanic modeling of fiber reinforced polymer shear-strengthened reinforced concrete beam

Shahriar Shahbazpanahi [a,b,*], Abang Abdullah Abang Ali [b], Alaleh Kamgar [a], Nima Farzadnia [b]

[a] *Department of Civil Engineering, Islamic Azad University, Sanandaj Branch, 66169, Pasdaran St., Sanandaj, Kurdistan, Iran*
[b] *Housing Research Center, University Putra Malaysia, 43300 Serdang, Malaysia*

ARTICLE INFO

Article history:
Received 17 March 2014
Received in revised form 7 August 2014
Accepted 20 August 2014
Available online 28 August 2014

Keywords:
A. Polymer–matrix composites (PMCs)
B. Fracture
C. Finite element analysis (FEA)

ABSTRACT

A numerical method is developed to model shear-strengthening of reinforced concrete beam by using fiber reinforced polymer (FRP) composites. Tensile crack is simulated by a non-linear spring element with softening behavior ahead of the crack tip to model the cohesive zone in concrete. A truss element is used, parallel to the spring element, to simulate the energy dissipation rate by the FRP. The strain energy release rate is calculated directly by using a virtual crack closure technique. It is observed that the length of the fracture process zone (FPZ) increases with the application of FRP shear-strengthening. The present model shows that the main diagonal crack is formed at the support in the control beam while it appears through the shear span in the shear-strengthened beam. Another important observation is that the load capacity increases with the number of CFRP sheets in the shear span.

1. Introduction

The first modeling of crack propagation in concrete was proposed by Hillerborg et al. [1]. It was shown that there is a region in front of the real crack tip, which leads to crack closure. It was called the fracture process zone (FPZ). The study of FPZ is vital to improve the fracture resistance of its member as well as crack propagation [2].

Hillerborg et al. [1] used cohesive stress to simulate FPZ in discrete cracks. In this model, as cohesive stress is a function of crack opening, this stress reaches tensile strength at the tip of the crack and reduces to zero at the critical opening of the crack. The amount of the area under stress-opening curve is equal to the energy release rate. This model, often referred to as the cohesive zone model (CZM), is deployed to model the FPZ in structures with normal sizes [3]. Either the nodal force release method or interface element with zero initial thickness technique is used for this purpose [4–6].

So far, the method suggested by Hillerborg et al. [1] has been applied more widely due to its practicality, accuracy, and cost-effectiveness [7]. Some studies have been carried out to improve the functionality of the CZM [8–11]. To model the CZM, two types of interface elements have been deployed. One of the most formal ones, [8,4] is the continuum cohesive zone model (CCZM) which uses the interface elements [12,9,10]. A different interface element is the discrete cohesive zone model (DCZM) which is very simple to use [13]. The DCZM reduces the computational time and is compatible with the finite element method [13]. The virtual crack closure technique (VCCT) is one of the DCZM methods that could calculate energy release rates directly and compare these rates to a critical value to study the crack propagation. In this technique, energy exerted to close the crack is calculated by multiplying the nodal force and displacement opening [14]. This approach is acceptable, simple, and powerful [5] and yet can be modified to study the crack propagation in concrete.

Fiber reinforced polymer (FRP) composites is increasingly used in concrete due to its corrosion resistance, low weight, high tensile strength and large strain [15]. The FRP is used to improve the flexural capacity and the shear strengthening as well as to prevent the crack growth [16–19]. FRP effect on tension cracks in concrete (Fig. 1) are essential in order to prevent the shear crack and enhance the shear-load bearing.

A few numerical models have been employed in FRP shear-strengthening with the fracture mechanics theory [20–24]. In studies that have previously been mentioned, the bond–slip attracts more attention than the cracks in concrete as well as the FRP ruptures [25–27]. These investigations are not influenced by the FRP shear strengthening on the FPZ, while the crack propagation

---

* Corresponding author at: Department of Civil Engineering, Islamic Azad University, Sanandaj Branch, 66169, Pasdaran St., Sanandaj, Kurdistan, Iran. Tel.: +98 9188710936.

E-mail addresses: sh.shahbazpanahi@iausdj.ac.ir, sh.shahbazpanahi@gmail.com (S. Shahbazpanahi).

http://dx.doi.org/10.1016/j.compositesb.2014.08.041
1359-8368/© 2014 Elsevier Ltd. All rights reserved.

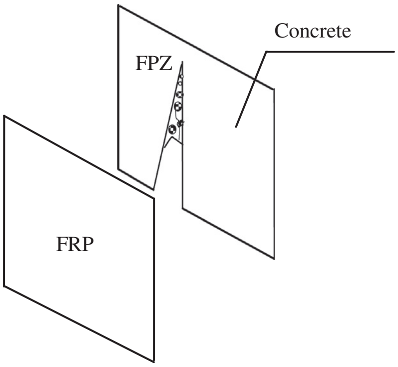

Fig. 1. The FPZ and shear-strengthened with FRP.

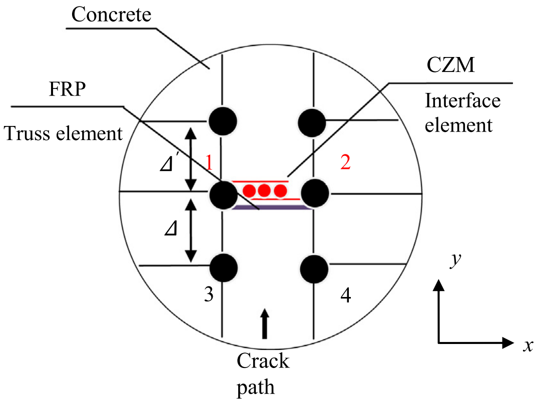

Fig. 2. Modeling crack and FRP shear strengthening.

criterion is taken into account. The effects of the FRP shear-strengthening on the crack propagation criterion have not been investigated based on the energy approach yet and this effect requires further study.

From the finite element point of view, the element stiffness should be properly chosen to model the FPZ in concrete. In practice, compared to the undamaged zones, the FPZ has a different stiffness. The different stiffness of the FPZ requires that energy be used in studying the crack growth. Therefore, it is important to use a suitable stiffness element to simulate the FPZ in the finite element method.

Also, to predict the crack propagation in composite materials, a correct estimate of the energy release rate is important. It is well-known that in non-linear fracture mechanics, the energy release rate determines whether the crack propagates or not. It is necessary to use accurate element stiffness and to consider the effect of the FRP on the crack propagation.

This paper focuses on the FPZ stiffness of the mixed-mode crack, modeling FRP shear strengthening, crack propagation criterion, and crack direction in shear-strengthened reinforced concrete beams. A non-linear spring element with softening behavior is proposed to model mix mode cracks, and the truss element is used parallel to the first one to simulate the linear behavior of shear-strengthened FRP. In this paper, concrete failures and FRP ruptures are more of interest in comparison with the slip and debonding of the FRP (Fig. 1). Another interface element is used to transfer shear in nodal forces between concrete and the FRP elements. The results are closer, in terms of the load–deflection and the crack path, to the test data, which have been reported by Adhikary and Mutsuyoshi [28], and by Soudki et al. [29].

2. Materials and methods

2.1. Interface element

Fig. 2 shows the interface element with a spring with softening behavior to model the CZM [5] and the linear elastic truss element to simulate the FRP parallel to the spring element at the crack tip between two nodes to calculate the internal forces. In the first iteration of the non-linear analysis, the node pairs 1–2 and 3–4 have the same coordinates and they are firmly linked together using a sufficiently large spring stiffness to guarantee that the crack is closed initially. A spring element is set at the crack tip between Nodes 1 and 2. Nodes 3 and 4 are dummy nodes and they are used only to illustrate the variation of the crack.

The element stiffness matrix and displacement vector for the element are given by:

$$\overline{\mathbf{K}} = \begin{bmatrix} \mathbf{K} & -\mathbf{K} \\ -\mathbf{K} & \mathbf{K} \end{bmatrix}, \quad \boldsymbol{u} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \tag{1}$$

where u_1 and u_2 are the displacement components for Node 1, while u_3 and u_4 are displacement components for Node 2 in the local system, respectively. The value of shear stiffness in the FRP transverse direction is assumed to be zero, hence in the local coordinate system, matrix **K** is defined by:

$$\mathbf{K} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} + E_F A / w_c \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{2}$$

where k_x and k_y are values of the stiffness of the CZM, w_c is the critical crack opening displacement in the stress-crack opening displacement (COD) curve, σ–COD curve (Fig. 3(a)), A is the FRP section area, and E_F is the elastic modulus for FRP (Fig. 3(b)). w_c is considered as the initial length of the FRP truss element, assuming that crack growth in concrete substrate is initially followed by the exposure of the FRP as the load increases.

2.2. Stiffness and constitutive model

In previous research, k_x and k_y are estimated from the Young's modulus and shear modulus of the concrete, respectively [13], while the stiffness of this zone, due to the interlock of aggregates, micro-cracks micro-failure, have to be changed to the stiffness of the softening zone. Therefore, it is essential to use an accurate stiffness for the interface element. To overcome this difficulty, the stiffness of the spring is improved based on the softening zone.

Fig. 3(a) shows bi-linear type σ–COD curve in concrete. The behavior initially is elastic and then it became softened. In the softening zone, there is still some resistance against opening, but

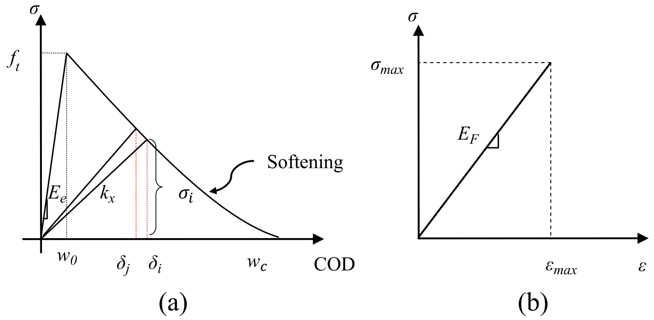

Fig. 3. (a) σ–COD curve of the concrete and (b) stress–strain of the FRP.