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Layer-wise and equivalent single layer models for smart multilayered plates

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ABSTRACT

Layer-wise and equivalent single layer plate models for magneto-electro-elastic multiphysics laminates are presented in a unified framework. They are based on variable kinematics and quasi-static behavior of the electromagnetic fields. The electromagnetic state of each single layer is preliminary determined by solving the corresponding governing equations coupled with the proper interface continuity and external boundary conditions. By so doing, the electromagnetic state is condensed into the plate kinematics and the layer governing equations are inferred by the principle of virtual displacements. This approach identifies effective mechanical layers, which are kinematically equivalent to the original smart layers. These effective layers are characterized by stiffness, inertia and load properties which take the multifield coupling effects into account as their definitions involve the electromagnetic coupling material properties. The layers governing equations are finally assembled enforcing the mechanical interface conditions. This allows to obtain the smart plate resolving system, which involves primary mechanical variables only. Results for thick simply-supported multilayered plates are obtained by an exact closed-form Navier-type solution and compared with benchmark 3*D* solutions to investigate the features and accuracy of the proposed modeling approach.

1. Introduction

The growing development and employment of smart materials open towards the possibility to make structures with multi-functional capabilities, related to the inherent coupling between mechanical and electrical fields, like in piezoelectrics, or among mechanical, electrical and magnetic fields, like in magneto-electro-elastic (MEE) materials. The effects of coupling among different physical fields can be successfully applied to a lot of technologies such as structural health monitoring, vibration control and energy harvesting, only to cite a few. In this framework, the use of multilayered and/or functionally graded structures appears very effective and reliable [1] and efficient modeling tools are then required for their analysis and design. Smart piezoelectric composite laminates and their modeling approaches received a lot of attention in the literature [2,3], whereas MEE materials and structures gained interest for their potential application only recently with the accompanying research activities on their modeling (e.g. [4–15]).

For composite laminates modeling and design, 2*D* plate theories have been developed with the aim of reducing the analysis effort preserving, as well, a suitable level of accuracy. These theories are classified into layer-wise (LW) and equivalent single layer (ESL) approaches [16]. The LW approach enables high accuracy with an associated computational cost that grows as the number of layers increases. On the other hand, ESL plate theories do not present this drawback as their solution complexity is independent from the number of layers resulting in more affordable analysis procedures; however they are generally less accurate than the layer-wise ones, especially for thick laminates whose reliable analysis generally demands higher order theories.

To formulate and implement both LW and ESL refined higher order theories for multilayered plates, Carrera proposed a powerful approach known as CUF (Carrera Unified Formulation) whose underlying ideas, principles and implementation issues for mechanical problems can be found in Refs. [17,18]. CUF offers a systematic procedure to generate different order refined plate models, considering the order of the theory as a free parameter of the formulation. The CUF was applied to smart laminates with both piezoelectric [19–25] and magneto-electro-elastic [26–28] layers.

Plate theories for multilayered smart laminates are generally formulated taking the electric and magnetic primary variables as independent state variable of the problem. More recently, Milazzo [29] proposed modeling of smart MEE laminates through the concept of an *effective mechanical plate* resulting from the condensation of the electromagnetic state into the mechanical variables,

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which provides for a resolving system involving kinematical variables only. By using this approach, a family of ESL models based on refined higher order plate theories has been derived through the CUF technique [30]. It is worth noting that this kind of modeling strategy shows appealing as it could take advantage of the well-established solution techniques and tools available for the mechanics of multi-layered plates. According with this rationale, the objective of the present work is to extend and generalize this approach to derive both LW and ESL refined models in a unified framework based on the CUF and highlight its features and limits with respect to the modeling assumptions.

The paper is organized as follows. The basic assumptions and governing equations are preliminary introduced in Section 2. Then, the model for single isolated smart layers with variable kinematics is formulated in Section 3 and in turn used to build the multilayered plate LW or ESL model as described in Section 4. In Section 5 closed form solution and results for simply-supported rectangular plates are presented and discussed. Finally, conclusions are drawn. Some definitions and derivations are presented in Appendices A and B to make the paper self-consistent and avoid making heavy the reading.

2. Governing equations and basic assumptions

Consider a multilayered plate referred to a coordinate system with the x_3 axis directed along the thickness and the x_1 and x_2 coordinates spanning the plate mid-plane Ω, whose boundary is denoted by $\partial\Omega$. The plate consists of N layers of homogeneous and orthotropic magneto-electro-elastic materials having poling direction parallel to the x_3-axis. Piezoelectric and elastic layers are obviously subcases of the more general magneto-electro-elastic case. The κth layer has constant thickness $t_\kappa = h_\kappa - h_{\kappa-1}$, being $h_{\kappa-1}$ and h_κ the x_3 coordinates of its bottom and top faces, respectively. The bottom and top surfaces of the plate are identified by $x_3 = h_l$ and $x_3 = h_u$, respectively. The plate is subjected to mechanical loads and to electric and magnetic actions applied on the top and bottom surfaces. For convenience' sake, a layer reference system is also introduced with a normalized thickness coordinate ζ_κ defined as

$$\zeta_k = \frac{2}{h_\kappa - h_{\kappa-1}} x_3 - \frac{h_\kappa + h_{\kappa-1}}{h_\kappa - h_{\kappa-1}} \tag{1}$$

2.1. Primary variables and gradient equations

As the elastic waves propagate several order of magnitude slower than the electromagnetic ones, the quasi-static approximation for the electromagnetic state is considered. Therefore, to describe the smart plate response the displacements $\boldsymbol{u} = \{u_1 \quad u_2 \quad u_3\}^T$, the electric potential Φ and the magnetic scalar potential Ψ are used as primary variables [31].

The strain field $\boldsymbol{\varepsilon}$ is suitably partitioned into the in-plane components $\boldsymbol{\varepsilon}_p = \{\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{12}\}^T$ and out-of-plane components $\boldsymbol{\varepsilon}_n = \{\varepsilon_{13} \quad \varepsilon_{23} \quad \varepsilon_{33}\}^T$ and, accordingly, the linear strain–displacements relationships read as

$$\boldsymbol{\varepsilon}_p = \mathcal{D}_p \boldsymbol{u} \tag{2a}$$

$$\boldsymbol{\varepsilon}_n = \mathcal{D}_n \boldsymbol{u} + \mathcal{D}_{x_3} \boldsymbol{u} \tag{2b}$$

where the following differential operators are introduced

$$\mathcal{D}_p = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_2} & 0 \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} & 0 \end{bmatrix} \tag{3a}$$

$$\mathcal{D}_n = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x_1} \\ 0 & 0 & \frac{\partial}{\partial x_2} \\ 0 & 0 & 0 \end{bmatrix} \tag{3b}$$

$$\mathcal{D}_{x_3} = \begin{bmatrix} \frac{\partial}{\partial x_3} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_3} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} \end{bmatrix} = \mathcal{D}_i \frac{\partial}{\partial x_3} \tag{3c}$$

being $\mathcal{D}_i$ the 3×3 identity matrix.

Also the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ are partitioned into their in-plane and out-of-plane components, which specify as $\boldsymbol{E}_p = \{E_1 \quad E_2\}^T, \boldsymbol{E}_n = \{E_3\}$, $\boldsymbol{H}_p = \{H_1 \quad H_2\}^T$ and $\boldsymbol{H}_n = \{H_3\}$. By introducing the following differential operators

$$\boldsymbol{\nabla}_p = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix} \tag{4a}$$

$$\boldsymbol{\nabla}_n = \begin{bmatrix} \frac{\partial}{\partial x_3} \end{bmatrix} \tag{4b}$$

the electric and magnetic gradient equations are written as

$$\boldsymbol{E}_p = -\boldsymbol{\nabla}_p \Phi \tag{5a}$$

$$\boldsymbol{E}_n = -\boldsymbol{\nabla}_n \Phi \tag{5b}$$

$$\boldsymbol{H}_p = -\boldsymbol{\nabla}_p \Psi \tag{6a}$$

$$\boldsymbol{H}_n = -\boldsymbol{\nabla}_n \Psi \tag{6b}$$

2.2. Constitutive equations

Consistently with the variables partition introduced in the previous section, the constitutive law for an orthotropic magneto-electro-elastic composite having electric and magnetic poling directions parallel to the x_3−axis is compactly written as

$$\begin{Bmatrix} \boldsymbol{\sigma}_p \\ \boldsymbol{\sigma}_n \\ \boldsymbol{D}_p \\ \boldsymbol{D}_n \\ \boldsymbol{B}_p \\ \boldsymbol{B}_n \end{Bmatrix} = \begin{bmatrix} \boldsymbol{C}_{pp} & \boldsymbol{C}_{pn} & \boldsymbol{0} & -\boldsymbol{e}_{np}^T & \boldsymbol{0} & -\boldsymbol{q}_{np}^T \\ \boldsymbol{C}_{np} & \boldsymbol{C}_{nn} & -\boldsymbol{e}_{pn}^T & -\boldsymbol{e}_{nn}^T & -\boldsymbol{q}_{pn}^T & -\boldsymbol{q}_{nn}^T \\ \boldsymbol{0} & \boldsymbol{e}_{pn} & \boldsymbol{\epsilon}_{pp} & \boldsymbol{0} & \boldsymbol{d}_{pp} & \boldsymbol{0} \\ \boldsymbol{e}_{np} & \boldsymbol{e}_{nn} & \boldsymbol{0} & \boldsymbol{\epsilon}_{nn} & \boldsymbol{0} & \boldsymbol{d}_{nn} \\ \boldsymbol{0} & \boldsymbol{q}_{pn} & \boldsymbol{d}_{pp} & \boldsymbol{0} & \boldsymbol{\mu}_{pp} & \boldsymbol{0} \\ \boldsymbol{q}_{np} & \boldsymbol{q}_{nn} & \boldsymbol{0} & \boldsymbol{d}_{nn} & \boldsymbol{0} & \boldsymbol{\mu}_{nn} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_p \\ \boldsymbol{\varepsilon}_n \\ \boldsymbol{E}_p \\ \boldsymbol{E}_n \\ \boldsymbol{H}_p \\ \boldsymbol{H}_n \end{Bmatrix} \tag{7}$$

where $\boldsymbol{C}_{ij}$ are matrices containing the elastic stiffness coefficients, the $\boldsymbol{\epsilon_{ij}}$ and $\boldsymbol{\mu_{ij}}$ matrices collect the dielectric constants and magnetic permeabilities, respectively, whereas $\boldsymbol{e}_{ij}$, $\boldsymbol{q}_{ij}$ and $\boldsymbol{d}_{ij}$ collect the piezoelectric, piezomagnetic and magnetoelectric coupling coefficients. Eventually, for convenience in the following manipulations, the matrices $\boldsymbol{e}_{pn}$ and $\boldsymbol{q}_{pn}$ are partitioned in their rows which are denoted by $\boldsymbol{e}_{pn_\gamma}$ and $\boldsymbol{q}_{pn_\gamma}$, respectively, being γ the row number. As we deal with multilayered plates the constitutive law matrices depend on the x_3 coordinate and in particular, for the paper scope, they are considered constant inside each layer.

2.3. Governing equations

The problem governing equations can be obtained by the extension of the Principle of Virtual Displacements (PVD) to magneto-electro-elastic structures, which states [31]

$$\int_V \left(\delta\boldsymbol{\varepsilon}_p^T\boldsymbol{\sigma}_p + \delta\boldsymbol{\varepsilon}_n^T\boldsymbol{\sigma}_n - \delta\boldsymbol{E}_p^T\boldsymbol{D}_p - \delta\boldsymbol{E}_n^T\boldsymbol{D}_n - \delta\boldsymbol{H}_p^T\boldsymbol{B}_p - \delta\boldsymbol{H}_n^T\boldsymbol{B}_n\right)dV$$
$$= \int_V \left(\delta\boldsymbol{u}^T\bar{\boldsymbol{f}} - \delta\Phi\bar{q}\right)dV - \int_V \rho\delta\boldsymbol{u}^T\ddot{\boldsymbol{u}}dV + \int_{\partial V}\left(\delta\boldsymbol{u}^T\bar{\boldsymbol{t}} - \delta\Phi\overline{Q}\right)d\partial V \tag{8}$$

where superimposed dots denote time derivatives, $\bar{\boldsymbol{t}}$ and $\bar{\boldsymbol{f}}$ are the applied surface tractions and body forces, $\overline{Q}$ and $\bar{q}$ are the surface and body electric charge density, ρ is the mass density and δ denotes virtual variations. By using Eqs. (2), (5) and (6) and