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A comprehensive study on the equivalent electrical conductivity tensor validity for thin multidirectional carbon fibre reinforced plastics

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ABSTRACT

The electrical conductivity of the dry carbon fibre multidirectional thin layers (without resin matrix material) can be expressed by an equivalent EC tensor. The flow of the electric current inside the material is depended, mainly, upon the material's microstructure and specifically upon the interlaminar microstructure. In the case of the multidirectional carbon fibre composites, two different methodologies were used in order to elucidate the EC behaviour of the multidirectional laminates both quantitatively and qualitatively. The first experimental setup was used in order to compare the measured electrical resistance to the calculated equivalent EC tensor. Also, the temperature field changes according to the equivalent EC tensor of each laminate and the second methodology was utilized in order to elucidate the validity of equivalent EC tensor indirectly, comparing the temperature field of the electro-thermal numerical models and the measured temperature field.

1. Introduction

Carbon fibre reinforced plastics (CFRPs) can be used in a plethora of applications due to the electrical properties of carbon fibres (CFs). Certain examples of CF applications, are among others; deicing of aircraft components, heated CFRP molds. Furthermore, in order to locate and assess damages, certain non-destructive control methods have been developed, such as electrical measurements [1]. Also, the use of the Joule effect as an innovative, non-destructive method that has been studied in order to identify interlaminar damages in CFRPs [2]. Additionally, CFRPs have found application in antennas [3] and transducers [4]. In all the aforementioned applications, the determination of the EC of the multilayered CFRPs is a basic prerequisite.

Carbon fibre reinforced plastics (CFRPs) are categorized into unidirectional laminates (UD) and multidirectional laminates. The electric conductivity (EC) of unidirectional CFRPs and unidirectional dry carbon fibre preforms has been thoroughly studied and is dependent upon the carbon fibre electrical conductivity, fibre direction [4–7] fibre volume fraction [8] as well as the temperature [9] and the plies' thickness [10].

In real structures, CFRPs are used in the form of multidirectional laminates. The determination of the EC of the anisotropic multilayered laminates is the most crucial factor for the calculation of the electric field and current density distribution. In different scientific fields the equivalent tensor for multilayer anisotropic materials has been used by Bruschke and Advani [11] in fluid mechanics (fluid flow through porous media) in order to calculate the equivalent permeability tensor of the multidirectional media. Xiao et al. [12] have reported a pure theoretical study using an equivalent EC tensor at the piezoresistivity and piezoconductivity of composites. Beche et al. [13] present a theoretical study in order to define a concept of effective (equivalent) thermoelectric tensors in idealized free-standing superlattices. All the aforementioned studies use an equivalent material tensor which is equal to the sum of each layer's EC tensor. Finally the authors have verified that the EC of the dry CF multidirectional preforms (porous carbon fibre multidirectional laminates) can be expressed by an equivalent second order tensor, which is derived from each layer's EC tensor [14]. This is valid (a) in the case where the plies' thickness is small relatively to the other dimensions neglecting the electrical gradient through the thickness of the material [15] and (b) the layers have perfect contact, namely there are no resin-rich areas between the layers (interlaminar and intralaminar homogeneity).

Concerning the thick composites, Wasselynck et al. [16] introduced an extra interface layer, the conductivity of which is obtained by averaging the tensors of the upper fibre layer, lower fibre layer. Also, Todoroki [17] presents a theoretical study for thick laminated CFRPs.

Continuing the research on the determination of the electrical properties in multidirectional media, we study thoroughly the

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validity of the equivalent EC tensor in the case of CFRPs. Firstly, we distinguished and discussed the cases were the CFRP material under consideration is in agreement with the idealized case. Then, two methodologies were used in order to validate the proposed equivalent EC tensor of the multidirectional CFRP laminates. The first one explores the accuracy of the equivalent EC tensor, by using direct resistance measurements for various stacking sequences of the CFRP laminates. The second one provides the verification of the equivalent EC tensor indirectly, by using the analytically calculated equivalent EC tensor into the coupled electrothermal problem. The electrothermal problem was solved numerically for rectangular domains using a finite difference method (FDM code) in MATLAB for given electrical boundary conditions. The calculated temperature field was compared against the measured with a thermal camera temperature field, for the same electrical boundary conditions, and for a plethora of representative stacking sequences.

All the aforementioned studies present exclusively theoretical results conserving the electrical conductivity of CFRPs, and none experimental. The validity of the equivalent EC tensor in the idealized case (very good contact of the layers) has been proved in the absence of polymer matrix in the case of dry CF multilayer materials (porous multidirectional media) [14].

In this study, the results prove both qualitatively and quantitatively the behaviour of the electric current in the case of thin multidirectional CFRPs (no electric gradient through the thickness) and in general case of interlaminar regions. A plethora of measurements and tests contributed to the validation of the conclusions, as if will be shown later on. The results and conclusions were drawn, having taken into consideration images from the optical microscope as well as materials of various microstructures.

2. Theory

2.1. Equivalent EC tensor of thin multidirectional laminates

Assuming that the electrically conductive body is homogeneous and thin (2D approach), the EC tensor can be easily determined (Appendix A). The EC tensor that describes the new material is composed of the EC tensors that describe each CF layer, separately. When the layers' thickness is relatively small compared to the other dimensions [15], then the EC can be expresses by an equivalent EC tensor, Eq. (1).

$$\boldsymbol{\sigma} = \frac{1}{S}\sum_{n=1}^{N} S_n \boldsymbol{\sigma}_n \Rightarrow \boldsymbol{\sigma} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{\sigma}_n \tag{1}$$

where (S) is the total cross section of the laminate, (S_n) is the cross section of each layer, ($\boldsymbol{\sigma}_n$) is the EC tensor of each layer and (N) is the number of plies of the multidirectional CFRP laminate considering plies of constant thickness. By combining plies of different fibre direction, a new material is produced, the EC of which depends upon its layers' electric conductivity. Fig. 1 shows the electrical conductivity ellipse of each ply of the laminate as well as the equivalent EC tensor of the laminate for four different stacking sequences.

The EC tensor of the multilayered medium can also be expressed as a function of temperature. In contrast to metals, the CF electric conductivity rises, as the material's temperature rises. As far as the thermal coefficient (α) is concerned, the value at fibre direction (0°) is different from the respective at the transverse direction ($a_0 \neq a_{90}$), Eq. (2).

$$\boldsymbol{\sigma}^T = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{\sigma}_n^T \Longleftrightarrow \boldsymbol{\sigma}^T = \frac{1}{N}\sum_{n=1}^{N} \left\{[m]_n([I] + [a]_n(T - T_o))[\rho]_n[m]_n^T\right\}^{-1} \tag{2}$$

where $[m]_n$ denotes the directional cosines and $[m]_n^T$ is the transpose matrix of the directional cosines, $[a]_n$ is the matrix of the thermal coefficients (linear dependence of resistivity) and $[\rho]_n$ is the VR tensor of each layer at principal directions.

2.2. Steady electric current in anisotropic continuous media

In the case of an anisotropic and electrical non-homogeneous medium the electrical potential field in 2D can be expressed by Eq. (3).

$$\begin{aligned}\nabla \cdot (\boldsymbol{\sigma} \cdot \nabla\phi) = 0 &\Longleftrightarrow (\nabla \cdot \boldsymbol{\sigma}) \cdot \nabla\boldsymbol{\phi} + \boldsymbol{\sigma} : \nabla(\nabla\phi) = 0 \\ &\Longleftrightarrow \frac{\partial\phi}{\partial x_1}\left(\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2}\right) + \frac{\partial\phi}{\partial x_2}\left(\frac{\partial\sigma_{21}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2}\right) \\ &\quad + \sigma_{11}\frac{\partial^2\phi}{\partial x_1^2} + 2\sigma_{12}\frac{\partial^2\phi}{\partial x_1\partial x_2} + \sigma_{22}\frac{\partial^2\phi}{\partial x_2^2} = 0\end{aligned} \tag{3}$$

where ($\mathbf{J}$) is the current density, ($\boldsymbol{\sigma}$) is the equivalent EC tensor of the multilayer material, (φ) is the electric potential field and (x_1, x_2) is the coordinate system. The symmetry of the EC tensor ($\sigma_{12} = \sigma_{21}$) is a consequence of the symmetry of the kinetic coefficients (Onsager theorem) [18].

In the case where the electrical conductivity is not a function of space (electrically homogenous medium) its value will be constant at all areas of the material. Therefore, the partial derivatives of the electric conductivity are equal to zero $\left(\frac{\partial\sigma_{ij}}{\partial x_i} = 0\right)$ and the electric potential field in anisotropic media can be expressed by a simplified elliptic partial differential equation. Since the EC tensor in anisotropic and homogeneous media is symmetric the electric potential field in 2D satisfies Eq. (4).

$$\begin{aligned}\nabla \cdot (\boldsymbol{\sigma} \cdot \nabla\phi) = 0 &\Longleftrightarrow \boldsymbol{\sigma} : \nabla(\nabla\phi) \\ &= 0 \overset{\sigma_{12}=\sigma_{21}}{\Longleftrightarrow} \sigma_{11}\frac{\partial^2\phi}{\partial x_1^2} + 2\sigma_{12}\frac{\partial^2\phi}{\partial x_1\partial x_2} + \sigma_{22}\frac{\partial^2\phi}{\partial x_2^2} = 0\end{aligned} \tag{4}$$

Eq. (4) demonstrates that the current density lines are not perpendicular to the equipotential lines, as in the well-known case of isotropic medium, Fig. 2. This is a significant difference between the electric behaviour of the anisotropic media and the isotropic one.

The typical boundary conditions for the general anisotropic medium are described by a combination of a first kind boundary condition at the constant electrically potential regions (Γ_d) and $\mathbf{J_n} = \mathbf{0}$ at electrically insulated regions (Γ_s). In the present work, we study thin orthogonal CFRPs. Fig. 3 provides schematically a representation of the problem solved for all the cases. The known electric potential has been applied between the boundaries (Γ_d) given by Eq. (5).

$$\begin{aligned} V &= f \quad \text{at} \quad x_1 = 0 \\ V &= 0 \quad \text{at} \quad x_1 = L \end{aligned} \tag{5}$$

The (Γ_s) boundaries are electrically insulated and there is no current flow in the normal to the boundary direction. The boundary conditions in this case are expressed by Eq. (6).

$$\begin{aligned} -\sigma_{11}\left(\frac{\partial\phi}{\partial x_1} + \frac{\sigma_{12}}{\sigma_{11}}\frac{\partial\phi}{\partial x_2}\right) = 0 \quad &\text{at} \quad x_1 = 0 \\ \sigma_{11}\left(\frac{\partial\phi}{\partial x_1} + \frac{\sigma_{12}}{\sigma_{11}}\frac{\partial\phi}{\partial x_2}\right) = 0 \quad &\text{at} \quad x_2 = W \end{aligned} \tag{6}$$

where (x_1, x_2) is the coordinate system, (ξ_1, ξ_2) is the coordinate system of the principle directions, (θ) denotes the angle between the horizontal direction and the principal direction of the EC tensor, (W) and (L) are the width and the length of the CFRP thin plate respectively.