Composites: Part B 67 (2014) 332–347

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Dynamic effective thickness in laminated-glass beams and plates

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ARTICLE INFO

Article history:
Received 14 November 2013
Received in revised form 17 June 2014
Accepted 14 July 2014
Available online 23 July 2014

Keywords:
A. Laminates
B. Viscoelasticity
D. Operational modal analysis
D. Mechanical testing

ABSTRACT

In recent years, several equations have been proposed to calculate deflections and stresses in laminated-glass beams and plates under static loading using the concept of effective thickness, which consists of calculating the thickness of a monolithic element with equivalent bending properties to a laminated element. Recently, an effective thickness for the dynamic behavior of laminated-glass beams has been proposed to enable the modal parameters (natural frequencies, loss factors and mode shapes) to be determined using an equivalent monolithic model. In the present paper, the technique has been extended to the two-dimensional case of rectangular laminated-glass plates and the steps needed to estimate the modal parameters of laminated-glass elements using this methodology are presented. The dynamic effective thickness concept has been validated by experimental tests made on a laminated-glass beam and a laminated-glass plate. The results show that good accuracy is achieved in the natural frequencies and mode shapes but high scatter is encountered in the loss factors.

1. Introduction

Laminated glass is a sandwich or layered material consisting of two or more plies of monolithic glass with one or more interlayers of a polymeric material. Thus, a laminated-glass element (beam, plate, etc.) is a composite material which combines the properties of the glass with the benefits of a highly elastic polymeric material, i.e. the structural behavior of laminated glass is that of a composite structure. The glass layers may be either normal annealed, heat-strengthened, chemically strengthened or tempered glass. When the cross-section is made of different glasses (e.g. one layer of annealed glass and the other of tempered glass) it is called a hybrid laminated-glass element. However, the treatments affect the ultimate strength but not the Young modulus, and therefore no distinction is made concerning the type of glass if the calculations are made prior to glass breakage. All polymeric interlayers are viscoelastic in nature [1], i.e. their mechanical properties are frequency (or time) and temperature dependent. Polyvinyl butyral (PVB) is the most commonly used interlayer material and is marketed in thicknesses of 0.38 mm or a multiple of this value (0.76 mm, 1.12 mm, 1.52 mm). However, the new ionoplastic interlayers improve the mechanical properties of laminated glass and maintains a significant advantage (higher stiffness and strength) over the PVB for a large range of temperatures [1]. This interlayer material is now in flat sheet form, in thicknesses of 0.89, 1.52, 2.27, and 3.05 mm, and as rolled sheeting, at 0.89 mm thickness. The simplest laminated-glass configuration consists of three layers: two monolithic glass plies and a polymeric core (see Fig. 1).

The response of laminated-glass elements varies between two borderlines [2]: (1) The layered limit corresponding to the case when the beam consists of free-sliding glass plies and (2) the monolithic limit, when the Euler–Bernoulli assumptions hold (plane sections remain plane) for the entire section of the laminated-glass element (the response of the composite beam approaches that of a homogeneous glass beam with an equal cross-section) [3,4]. As the tensile modulus of the PVB is far less in comparison with that corresponding to glass, significant transverse shear appears in the viscoelastic layer [1,8,10].

In the analytical and numerical models, glass mechanical behavior is usually modeled as linear-elastic prior to glass breakage, whereas the polymeric interlayer is characterized as linear-viscoelastic. Laminated glass is easy to assemble in a finite-element model but many small 3D elements are needed to mesh accurately, which, on the other hand, are very high time consuming. In the last few years, some papers have been published on the calculation of laminated-glass elements, examining the concept of effective thickness [1,3,4,8]. The method consists of calculating the thickness of a monolithic element with bending properties equivalent to those of the laminated one. The effective thickness can then be used in analytical equations and simplified finite-element models instead of the laminated-glass element [9].

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http://dx.doi.org/10.1016/j.compositesb.2014.07.018
1359-8368/

Nomenclature

D	flexural stiffness in plates
E	Young modulus
E_{eff}	effective Young modulus
E_1	Young's modulus of glass layer 1
E_3	Young's modulus of glass layer 3
EI^*	complex flexural stiffness in beams
$E_2^*(\omega)$	complex tensile modulus for the polymeric interlayer
$E'(\omega)$	real component of the tensile complex modulus (storage)
$E''(\omega)$	imaginary component of the tensile complex modulus (loss)
EI_T	$E_1I_1 + E_3I_3$
$E_2(t)$	viscoelastic relaxation tensile modulus for polymeric interlayer
E_∞^2	equilibrium tensile modulus for the polymeric interlayer
$G_2(t)$	viscoelastic relaxation shear modulus for the polymeric interlayer
$G_2^*(\omega)$	complex shear modulus for the polymeric interlayer
G_0	glassy shear modulus
H_1	thickness of glass layer 1 in laminated glass
H_2	thickness of polymeric layer in laminated glass
H_3	thickness of glass layer 3 in laminated glass
H_0	$H_2 + \left(\frac{H_1+H_3}{2}\right)$
I	second moment of area
I_1	$\frac{H_1^3}{12}$
I_3	$\frac{H_3^3}{12}$
I_T	$I_1 + I_3 = \frac{H_1^3+H_3^3}{12}$
$K(t)$	viscoelastic bulk modulus
L	length of a glass beam
T	temperature
T_0	reference temperature
Y	$\frac{H_0^2E_1H_1E_3H_3}{EI_T(E_1H_1+E_3H_3)}$
Y_1	$\frac{12H_0^2H_1H_3}{(H_1^3+H_3^3)(H_1+H_3)}$

Lowercase letters

a_T	shift factor
b	width of a glass beam
e_i	modulus coefficient in Prony's series viscoelastic model
$g(x)$	shape function (Galuppi and Royer Carfagni model)
g_M^*	shear parameter M&Ms model
g_R^*	shear parameter RKU model
i	imaginary unit
k_I	wavenumber
k_M^*	complex wave number
$\bar{m}$	mass per unit area
t	time
w	deflection

Greek symbols

Ω^*	non-dimensional complex frequency
β	bucking ratio for a beam
ζ	modal damping ratio
η	loss factor
η_2	loss factor of the polymeric interlayer of laminated glass
ν_i	Poisson ratio of the i-th glass layer
ρ_i	mass density of the i-th glass layer
τ_i	time coefficient in Prony's series viscoelastic model
ω	frequency

The aim of the present paper is to propose a simplified method to estimate the modal parameters of rectangular laminated-glass plates while avoiding the use of finite-element models or complicated analytical models. The method is based on the dynamic effective thickness proposed in a previous paper [9] for laminated-glass beams, which is here extended to the two-dimensional case of rectangular laminated-glass plates. An alternative to the effective thickness is the concept of effective Young modulus, which can be used interchangeably for laminated-glass elements with the same accuracy. This technique can be applied to three-layered laminated-glass plates with glass showing a linear elastic behavior and the polymeric core showing viscoelastic behavior. Thus the glass layers can be made of different types of glass (annealed, tempered, heat-strengthened, etc.), and the traditional cores (PVB, ionoplastic, etc.) can be considered in this model. In this paper, the modal parameters (natural frequencies, loss factors, and mode shapes) of a $1400 \times 1000 \times 16$ mm laminated-glass plate pin-supported at the four corners, and of a beam 1 m long and 12 mm thick, both the beam and the plate with annealed glass plies and PVB core, were estimated using the effective thickness concept. For the validation of the model, operational modal tests were performed on the beam and the plate, and the modal parameters identified from the experimental responses were compared with those predicted using the effective thickness concept.

2. State of the art

2.1. Viscoelastic behavior

The mechanical properties of a linear-viscoelastic material are frequency (or time) and temperature dependent [11]. In the frequency domain, the complex tensile modulus, $E_2^*(\omega)$, at temperature T is given by:

$$E_2^*(\omega,T) = E_2'(\omega,T) + i \cdot E_2''(\omega,T) = E_2'(\omega,T)(1 + i \cdot \eta_2(\omega,T)) \quad (1)$$

where superscript '$*$' indicates complex, ω represents the frequency, i is the imaginary unit, $E_2'(\omega,T)$ and $E_2''(\omega,T)$ are the storage and the loss tensile moduli, respectively, and

$$\eta_2(\omega) = \frac{E_2''(\omega,T)}{E_2'(\omega,T)} \quad (2)$$

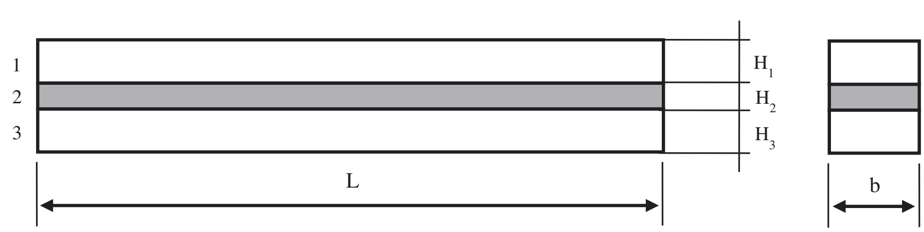

Fig. 1. Laminated glass.