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Vibrational analysis of advanced composite plates resting on elastic foundation

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ABSTRACT

This paper presents a free vibration analysis of functionally graded plates (FGPs) resting on elastic foundation. The displacement field is based on a novel non-polynomial higher order shear deformation theory (HSDT). The elastic foundation follows the Pasternak (two-parameter) mathematical model. The governing equations are obtained through the Hamilton's principle. These equations are then solved via Navier-type, closed form solutions. The fundamental frequencies are found by solving the eigenvalue problem. The degree of precision of the current solution can be noticed by comparing it with the 3D and other closed form solutions available in the literature.

1. Introduction

Functionally graded materials (FGMs) are a type of heterogeneous composite material in which the properties change gradually over one or more directions. This material is produced by mixing two or more materials in a certain volume ratio. Material properties of FGMs vary along the material size depending on a function. FGMs have been proposed, developed and successfully used in industrial applications since 1980s [1]. Nowadays, FGMs are an alternative materials widely used in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical and shipbuilding industries.

Classical composites structures such as fiber reinforced plastic (FRP) suffer from discontinuity of material properties at the interface of the layers and constituents. Therefore the stress fields in these regions create interface problems and thermal stress concentrations under high temperature environments. Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material [2]. These problems can be decreased by gradually changing the volume fraction of constituent materials and tailoring the material for the desired application.

Because of the widespread applications of foundations in engineering, several models to describe the mechanical behavior of elastic foundations were successfully formulated. Among them, a one-parameter model to describe the mechanical behavior of elastic foundations was discussed by Winkler [3], whereas Pasternak [4] presented a two-parameter model, which considers the shear deformation between the springs over the one-parameter model. The Winkler model can be considered a special case of Pasternak model by setting the shear modulus to zero. However, not many works on composites on elastic foundation based on HSDT exist.

Regarding the dynamic behavior of FGMs, many papers have been published recently. Leissa [5] presented 3D exact solution for the free vibration analysis of FGPs. Carrera [6] presented the free vibration analyses of layered plates, cylindrical and spherical shells made of isotropic and orthotropic layers for simply supported boundary condition. The transverse normal stress effects were included in the displacement model by allowing different polynomial orders. Liu and Liew [7] analyzed free vibration of rectangular plates with mixed boundary conditions on the basis of the first order shear deformation theory (FSDT). Matsunaga [8] investigated a two-dimensional polynomial HSDT for analyzing thick simply supported rectangular plates resting on elastic foundations.

Vel and Batra [2] developed a 3D exact solution for free and forced vibrations of simply supported FGPs. Lam et al. [9] used Green's functions and presented canonical exact solutions for bending, buckling and vibration of Levy-type plates resting on elastic foundation. Zhou et al. [10] were based on a three dimensional Ritz method with Chebyshev polynomials. Qian et al. [11] studied the static and dynamic deformation of thick FGPs. The author used an HSDT solved by meshless local Petrov–Galerkin

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method. Malekzadeh and Karami [12] studied the free vibration of rectangular plates of continuously varying thickness on two-parameter elastic foundations by using the differential quadrature method (DQM).

Batra and Jin [13] considered FGPs which were obtained by changing the fiber orientation. Free vibration results were provided by using the finite element method. Shufrin and Eisenberger [14] presented a numerical calculation of the natural frequencies and buckling loads for thick elastic rectangular plates with various combinations of boundary conditions using HSDT. Ferreira et al. [15] analyzed the free vibration of FGPs based on the FSDT and HSDT using the Mori–Tanaka homogenization method and the global collocation method with multiquadratic radial basis functions. Shimpi and Patel [16], based on the interesting work on the 2-unknowns plate theory, studied the free vibrations of orthotropic plates.

Uymaz and Aydogdu [17] also developed a 3D vibration solution for FGPs. Huang et al. [18] investigated the benchmark solutions for thick FGPs resting on Winkler–Pasternak elastic foundations using the 3D elasticity theory. Nagino et al. [19] presented 3D free vibration analysis of isotropic rectangular plates with any thicknesses and arbitrary boundary conditions using the B-spline Ritz method based on the elasticity theory.

Lu et al. [20], based on the 3D elasticity theory, studied the free vibration analysis of FG thick plates resting on elastic foundation. Zhao et al. [21] presented a free vibration analysis of FGPs by using the element-free kp-Ritz method. Malekzadeh [22] investigated free vibration analysis of thick FGPs resting on elastic foundations based on 3D elasticity theory using the DQM. Talha and Singh [23] investigated the static and free vibration analysis of FGPs by using the finite element method (FEM) and a polynomial HSDT. Hosseini-Hashemi et al. [24] presented an exact closed form Levy-type solution based on the Reddy's HSDT.

New non-polynomial HSDTs for classical and advanced composite plates and shells were developed by Mantari et al. [25,26] and Mantari and Guedes Soares [27]. Navier-type, closed form solutions were provided for the static and free vibration analysis of simply supported boundary conditions. Neves et al. [28,29], presented a sinusoidal and a hybrid type quasi-3D hyperbolic shear deformation theory for static and free vibration analysis of FGPs. Sheikholeslami and Saidi [30] studied the free vibration analysis of FGPs resting on two-parameter elastic foundation using a HSDT and an analytical approach. The authors expanded the displacement components in the thickness direction using the Legendre polynomials. Mechab et al. [31] considered the static and dynamic analysis of FGPs with new non-polynomial shear strain shape function (hyperbolic). Thai and Kim [32] developed a HSDT of four unknowns for bending and free vibration analysis of FGPs.

Jin et al. [33] presented a 3D exact solution for the free vibrations of thick FGPs with general boundary conditions. Akavci [34] presented a free vibration analysis of FGPs on elastic foundation applying a non-polynomial HSDT and an optimization procedure.

In the present paper, the free vibration analysis of FGPs resting on elastic foundations is studied. This novel non-polynomial HSDT accounts for adequate distribution of the transverse shear stresses through the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, thus a shear correction factor is not required. The mechanical properties of the plates are assumed to vary in the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. The governing equations of a type of FGPs resting on elastic foundation are derived by employing the Hamilton's principle. These motion equations are then solved via Navier solution. As a result, fundamental frequencies are found by solving eigenvalue problem. The accuracy of the present code is verified by comparing it with HSDT's solutions available in literature.

2. Theoretical formulation

2.1. Functionally graded plates

A rectangular plate of uniform thickness "h", length "a", and the width "b", made of a FGM and resting on elastic foundation is shown in Fig. 1. The rectangular Cartesian coordinate system x, y, z, has the plane $z = 0$, coinciding with the mid-surface of the plate. The material properties vary through the thickness with a power law distribution, which is given below (see Fig. 2):

$$\begin{aligned} &P_{(z)} = (P_t - P_b)V_{(z)} + P_b, \\ &V_{(Z)} = \left(\frac{z}{h} + \frac{1}{2}\right)^p, \\ &-\frac{h}{2} \leqslant z \leqslant \frac{h}{2} \end{aligned} \tag{1a-c}$$

where P denotes the effective material property, P_t and P_b denote the property of the top and bottom faces of the plate, respectively, and "p" is the exponent that specifies the material variation profile through the thickness. The effective material properties of the plate, including Young's modulus, E, and shear modulus, G, vary according to Eq. (1a,b), and Poisson ratio, "v" is assumed to be constant.

2.2. Displacement base field

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point (x, y, $\pm h/2$) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows:

$$\begin{aligned} &\bar{u}(x,y,z) = u(x,y) + z\left[y^*\theta_1 - \frac{\partial w}{\partial x}\right] + f(z)\theta_1, \\ &\bar{v}(x,y,z) = v(x,y) + z\left[y^*\theta_2 - \frac{\partial w}{\partial y}\right] + f(z)\theta_2, \\ &\bar{w}(x,y,z) = w, \end{aligned} \tag{2a-c}$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$, are displacements in the x, y, z directions, u, v and w are mid-plane displacements, θ_1 and θ_2 are rotations of normal to the mid-plane about y- and x-axis. u, v, w, θ_1 and θ_2 are the five unknown displacement functions of mid-plane of the plate, whilst $y^* = -f'\left(\frac{h}{2}\right)$. $f(z)$ represents the shear strain shape function for determining the appropriate distributions of the transverse shear strains and stresses along the thickness and given as:

$$f(z) = ze^{m\cos(nz/h)} \tag{3}$$

The appropriate value of the parameters "m" and "n" plays an important role in the accuracy of the present HSDT, and consequently need to be calculated. As usual, when proposing a new HSDT, it is important to obtain close to 3D exact solutions. The selection of these parameters will be discussed in the numerical results section.

2.3. Kinematic relations and constitutive relations

In the derivation of the necessary equations, small strains are assumed (i.e., displacements and rotations are small, and obey Hooke's law). The linear strain expressions derived from the displacement model of Eqs. (2a–c), valid for thin, moderately thick and thick plate under consideration are as follows: