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Finite element analysis of free vibration of the delaminated composite plate with variable kinematic multilayered plate elements

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ABSTRACT

Composite laminates are prone to delamination. Implementation of delamination in the Carrera Unified Formulation frame work using nine noded quadrilateral MITC9 element is discussed in this article. MITC9 element is devoid of shear locking and membrane locking. Delaminated as well as healthy structure is analyzed for free mode vibration. The results from the present work are compared with the available experimental or/and research article or/and the three dimensional finite element simulations. The effect of different kinds and different percentages of area of delamination on the first three natural frequencies of the structure is discussed. The presence of open-mode delamination mode shape for large delaminations within the first three natural frequencies is discussed. Also, the switching of places between the second bending mode, with that of the first torsional mode frequency is discussed. Results obtained from different ordered theories are compared in the presence of delamination. Advantage of layerwise theories as compared to equivalent single layer theories for very large delaminations is stated. The effect of different kinds of delamination and their effect on the second bending and first torsional mode shape is discussed.

1. Introduction

Composite laminate structures are widely used in aircraft, helicopters, wind turbine blades and in other industries. There are many ways of analyzing laminated structures; a detailed review articles on available methods for the analysis of laminated plates and shells, and its historical development are documented in articles [1,2]. Laminated composite structures are prone to defects such as matrix cracking, strength and stiffness degradation due to aging or corrosion, and delamination between plies [3,4]. Delamination in the composite structure may occur either during the manufacturing process or during service period of the structure [5]. However, delamination in a structure may lead to catastrophic failure of the structure [3,6,7]. Delamination models are required to facilitate the understanding of the effect of the delamination on the structures, and analyze possible algorithms for structural health monitoring of delaminated structures.

Analysis of laminated structures can be carried out using many theories. Available theories of composite laminated plates can be classified into [8,9]:

- Equivalent single layer (ESL) theories.
- Layerwise (LW) theories.
- Continuum-based three dimensional (3D) elasticity theories.

ESL theories can be grouped into axiomatic or asymptotic based theories, depending on the method of derivation [8]. In axiomatic framework, displacement or/and stress in the thickness direction is assumed and the 3D physical problem is collapsed into a two dimensional mathematical problem [8]. Based on the order of displacement approximation in the thickness direction, ESL theories are further classified into Classical Laminated plated Theory (CLT), First order Shear Deformation Theory (FSDT), Higher order Shear Deformation Theories (HSDT), and Zig-Zag (ZZ) theory. In asymptotic based theory, 3D energy terms are expanded in terms small parameters. These small parameters may be geometrical parameters, like ratio of thickness to the wavelength of deformation, maximum allowable strains, or/and material small parameters like shear modulus to Young's modulus ratio. Three dimensional energy terms are expanded as a series in these small

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parameters and, based on requirements, higher order terms are truncated, in the variational statement [10]. Layerwise theories are kind of quasi 3D theories, though each layer is analyzed using an ESL theory. Again, based on the independent variables chosen, theories can regrouped into displacement-based, stress-based or mixed formulations.

Different refined and advanced shell/plate models are contained in the Carrera's Unified Formulation (CUF). The CUF permits to obtain, in a general and unified manner, several models that can differ by the chosen order of expansion in the thickness direction, by the equivalent single layer or layer wise approach and by the variational statement used [11,12]. By implementing delamination model in the CUF frame-work, analysis of delaminated structures can be carried out using several models.

Finite element implementation of above theories is carried out by many researchers. Finite element implementation of ESL theories can be broadly classified based on:

- shape of element (Triangular or Quadrilateral) [13];
- number of nodes in the element [14];
- degrees of freedom per node [15,9];
- type of interpolation function [16,9]; and
- integration technique [17].

A comprehensive review on the finite element implementation of plate theories is given in the article by [9]. A comparative view on ESL and LW plate theories is give by [18]. Layerwise plate theory is used by many authors and for different application; few of the references for LW plate theories are [19–21]. Trigonometric LW shear deformation theory is implemented by [22]. [23] introduced Zig-Zag (ZZ) plate theory for laminated plates. ZZ plate theory is extended to include higher-order thickness function by [24]. ZZ plate theories are used in many applications, and some examples are given by these articles [25–27].

Delamination in the plates is modeled by many researchers, and delamination modeling can be broadly categorized into (a) the region approach and (b) the layerwise approach [28]. In region-wise approach, delaminated segment is divided into sub-laminates and the continuity conditions are imposed at the junctions of delaminated segments and the healthy segment. In layerwise theories, delamination can be modeled by introducing discontinuity functions in the displacement fields or by adding an additional embedded layer at the interface of delamination [28,29,5]. Delamination models can also be regrouped into free-mode delamination models and constrained-mode delamination models [28]. In free-mode delamination models, the sub-laminates of the delaminated segments are allowed to move independently without touching each other. Free-mode delamination model leads to non-physical displacement modes. Constrained-mode delamination models will not allow such non-physical displacement modes. Constrained-mode delamination models impose penalty conditions on the delaminated sub-laminates, by restricting the sub-laminates to move together. However, this option would not simulate the open-mode deformation conditions. This led to further enhancement of the constrained-mode delamination model by incorporating nonlinear springs in between the delaminated sub-laminates. A detailed review and historical development of delamination models is given by authors Della and Shu [28]. Delamination models are implemented in ZZ theories by [25], and by [30].

The present article uses Mixed Interpolation Tensorial Component, nine node quadrilateral (MITC9) element for the laminated plate analysis. MITC9 element is devoid of shear and membrane locking phenomenon [31]. Delamination model has been implemented in CUF frame-work. Results for the first few natural frequencies and modeshapes of the structure are obtained using CUF plate code for healthy as well as delaminated plate. Results are compared with existing literature or with 3D finite element simulation. The major contribution of the present article is the implementation of the delamination model in the CUF frame-work. The results for the delaminated plates for higher order theories, other than those presented in the literature are tabulated.

2. Variable kinematic model via Carrera Unified Formulation

Carrera Unified Formulation (CUF) is a technique which handles a large variety of bi-dimensional models in an unified manner. According to CUF, the governing equations are written in terms of few fundamental nuclei which do not formally depend on the order of expansion N used in the z direction and on the description of variables: ESL or LW. The application of a two-dimensional method for plates permits to express the unknown variables as a set of thickness functions depending only on the thickness coordinate z and the corresponding variables depending on the in plane coordinates x and y. So that, a generic variable, for instance the displacement $u(x,y,z)$, and its variation $\delta u(x,y,z)$, are written according to the following general expansion:

$$\boldsymbol{u}(x,y,z)=F_s(z)\boldsymbol{u}_s(x,y),\quad \delta\boldsymbol{u}(x,y,z)=F_\tau(z)\delta\boldsymbol{u}_\tau(x,y),\quad \text{with}\quad \tau,s=0,\ldots,N \tag{1}$$

Bold letters denote arrays and the summing convention with repeated indexes τ and s is assumed. The order of expansion N goes from first to higher-order values and, depending on the used thickness functions, a model can be ESL or LW. If the variable is assumed for the whole multilayer, the approach is ESL and a Taylor expansion is employed as thickness function $F(z)$:

$$\boldsymbol{u}=F_0\,\boldsymbol{u}_0+F_1\,\boldsymbol{u}_1+\ldots+F_N\,\boldsymbol{u}_N=F_\tau\,\boldsymbol{u}_\tau \text{ with } \tau=0,1,\ldots,N \tag{2}$$

$$F_0=z^0=1,\ F_1=z^1=z,\ \ldots,\ F_N=z^N \tag{3}$$

When the description is LW the variable is considered independent in each layer:

$$\boldsymbol{u}^{\boldsymbol{k}}=F_t\,\boldsymbol{u}_t^k+F_b\,\boldsymbol{u}_b^k+F_l\,\boldsymbol{u}_l^k\quad with\quad l=2,\ldots,N \tag{4}$$

where t and b indicate the top and bottom of the plate and the thickness functions $F(z)$ are combinations of Legendre polynomials:

$$P_0=1,\ P_1=\zeta_k,\ P_2=\frac{(3\zeta_k{}^2-1)}{2},\ P_3=\frac{5\zeta_k{}^3}{2}-\frac{3\zeta_k}{2},\ P_4=\frac{35\zeta_k{}^4}{8}-\frac{15\zeta_k{}^2}{4}+\frac{3}{8} \tag{5}$$

$$F_t=\frac{P_0+P_1}{2},\ F_b=\frac{P_0-P_1}{2},\ F_l=P_l-P_{l-2}\quad with\quad l=2,\ldots,N \tag{6}$$

The chosen functions have the following interesting properties:

$$\begin{aligned}\zeta_k&=1\ :\ F_t=1;\ F_b=0;\ F_l=0\quad at\ top\\ \zeta_k&=-1:\ F_t=0;\ F_b=1;\ F_l=0\quad at\ bottom\end{aligned} \tag{7}$$

that is the interface values of the variables are considered as unknowns.

It is possible to obtain the FSDT model [32,33] from an ESL theory with first order of expansion, by considering a constant transverse displacement through the thickness. An appropriate application of penalty techniques to shear strains leads to CLT [34].