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Enhanced Effective Thickness of multi-layered laminated glass

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ABSTRACT

The stiffness and strength of laminated glass, a composite of glass layers bonded together by polymeric interlayers, depends upon shear coupling between the glass plies through the polymer. In the design practice, this effect is commonly considered by defining the effective thickness of laminated glass, i.e., the thickness of a monolith with equivalent bending properties. Various theories have been proposed to calculate such a value for a package of two layers of glass and one polymeric interlayer, but extrapolation to a higher number of layers gives in general inaccurate results. Here, the *Enhanced Effective Thickness* method, previously proposed for two-glass-layer composites, is extended to the case of laminated glass beams made (i) by three layers of glass of arbitrary thickness, or (ii) by an arbitrary number of equally-thick glass layers. Comparisons with numerical experiments confirm the accuracy of the proposed approach also in these cases.

1. Introduction

Laminated glass is a composite widely used in civil engineering, as well as in automotive, aeronautics and shipbuilding, thanks to its transparency, strength and other advantageous aspects, such as sound-insulation capability and non-catastrophic post-glass-breakage response. Its traditional use as an infill panel is the most popular, but an alternative structural use has emerged in which glass elements contribute to the overall load bearing capacity of the structure or sub-structure [1]. Laminated glass is typically made of two glass plies bonded by a thermoplastic polymeric interlayer with a treatment in autoclave at high pressure and temperature. This process induces a strong chemical bond between the materials, due to the union between the hydroxyl groups along the polymer and the silanol groups on the glass surface. Through lamination, safety in the post-glass-breakage phase is increased because fragments remain attached to the interlayer; risk of injuries is reduced and broken glass maintains a certain cohesion that prevents catastrophic collapse.

In the pre-glass-breakage phase, the polymeric interlayers are too soft to present flexural stiffness *per se*, but they can provide shear stresses that constrain the relative sliding of the glass plies [2,3]. Of course, the degree of coupling of the glass layers depends upon the shear stiffness of the interlayer [4]. Thus, the flexural performance is somehow intermediate between the two borderline cases [5,6] of (i) *monolithic limit*, with perfect bonding between glass plies (shear-rigid interlayers) and (ii) *layered limit*, with frictionless sliding glass plies. Since stress and strain are much lower in the monolithic than in the layered limit, to avoid redundant design a large number of theoretical studies have been devoted, also in recent years, to this subject [7–9], with a wealth of experimental activity [10–12].

In general, the modeling of composite laminated structures with a "soft" core is one of the most active research fields of the last decades, since an accurate stress analysis is required to design structural parts. Hence, several theories have been developed to describe the structural behavior of sandwich beam [13,14]. In particular, the well-know First-Order Shear Deformation approach [15], based on the assumption that planes normal to the midplane remain straight, but not necessarily normal to it, after deformation has been followed by many authors in last decades (see, among others, [16–18]). This theory usually provides good results in terms of maximum displacement under appropriate choice of the shearing rigidity.

Nevertheless, the precise calculation of the coupling offered by the interlayer is quite difficult and usually requires numerical analysis, complicated by the fact that response of the polymer is nonlinear, viscoelastic and temperature dependent. A common practice is to consider the polymer as linear elastic, accounting for its viscoelasticity through an equivalent elastic modulus, assumed equal to the relaxed modulus under constant strain after a time comparable with the characteristic duration of the design action.

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In numerical computations, laminated glass may be modeled with layered shell elements that take into account the competing stiffness between glass and interlayer, but not all the libraries of commercial numerical codes contain such elements; furthermore, a full three-dimensional analysis is complicated and time consuming. This is why, in the practice and especially in the preliminary design, it is useful to rely upon simple methods. The most common approach consists in defining the so called *effective thickness*, i.e., the thickness of a glass monolith with bending properties equivalent to the laminated element. More precisely, the effective thickness of a laminated glass plate is the (constant) thickness of a monolithic plate that, under the same boundary and load conditions, presents the same maximal stress or maximal deflection. This is a very practical definition, but the literature and the technical standards record various conflicting formulas for its quantification.

The most common approaches are the one prescribed by the Project of European Norm prEN 16612 [19], which inherits the former prEN 13474 [20], and the one recorded in ASTM-E1300 [21] following the proposal by Bennison and Stelzer [22,23] and the original work by Wölfel [24]. The first formulation condenses all the effects of the interlayer in one coefficient ω, which depends upon the shear stiffness family of the interlayer, which is defined by the Project of European Norm prEN 16613 [25]. This method is apparently very simple because its relies upon a linear interpolation between the layered and the monolithic limit but, as widely discussed in [26], its basis has to be questioned on a theoretical ground. In fact, the method of prEN 16612 does not take into account the important roles played by the load type and by the size-effect (length, width and package composition) on the shear-coupling effects of the interlayer. Furthermore, as it will be demonstrated later on in Sections 4.2 and 4.3, results are not reliable when this method is extended to multilaminates. The method of ASTM E1300, as discussed in [27,28], gives excellent results for cases in which the deformation of the plate tends to be cylindrical and, in particular, for simply supported laminated beams under uniformly distributed loading, but is inaccurate in other cases.

An alternative method [27], called *Enhanced Effective Thickness*, has been recently proposed by the authors. This is based upon a variational approach where, through minimization of the strain energy functional, the best approximation for the response of laminated glass is selected among a restricted class of shape functions for the deflection surface. The main underlying hypotheses are: (i) the interlayer has no axial or bending stiffness, but only shear stiffness; (ii) shear strain of glass is negligible; (iii) both glass and polymer are linear elastic materials; and (iv) geometric non-linearities are ineffective. Remarkably the method, originally conceived of for beams under bending [27], can be naturally extended to the two-dimensional case of plates [28] under the most various load and boundary conditions [29,30].

In general, all the aforementioned methods have been formulated for laminates composed by two glass layers bonded by one interlayer. Attempts have been made to extend the prEN 16612 and the ASTM E1300 methods to the case of three or more glass plies, but the accuracy is in general not satisfactory, as it will be shown later on in Section 4. The purpose of this article is to show that the Enhanced Effective Thickness method can be naturally extended to the case of multilayered laminated glass beams. More specifically, the cases that will be treated here are laminated beams composed either by three glass layers of arbitrary thickness, or by an arbitrary number of equally-thick glass layers. By defining an effective moment of inertia of the composed beam as the weighted *harmonic* mean of the moments of inertia corresponding to the layered and monolithic limit, practical formulas for the *stress*- and *deflection-effective thickness* are proposed. The method covers various boundary and loading conditions. Comparisons with numerical experiments highlight the much higher accuracy of the proposed approach with respect to the other formulations.

2. Five-layered beams with glass plies of arbitrary thickness

With reference to the system (x, y) of Fig. 1, consider the laminated beam of length l and width b, composed of three glass plies, of thickness h_1, h_2 and h_3 and Young's modulus E, bonded by thin polymeric interlayers (of thickness t_1 and t_2 respectively), with shear modulus G. The beam is loaded by an arbitrary load per unit length $p(x)$, not necessarily uniformly distributed. Let us define

$$A_i = h_i b, \quad I_i = \frac{bh_i^3}{12} \ (i = 1, \ldots, 3), \quad H_1 = t_1 + \frac{h_1 + h_2}{2},$$
$$H_2 = t_2 + \frac{h_2 + h_3}{2}. \tag{2.1}$$

2.1. The model

Under the hypotheses that the glass–polymer bond is perfect and the interlayer strain in direction y is negligible, provided that strains are small and rotations moderate, the kinematics is completely described by the vertical displacement $v(x)$, the same for the three glass components, and the horizontal displacements $u_1(x)$, $u_2(x)$ and $u_3(x)$ of the centroid of the cross-sectional areas of glass plies. Following the same procedure of [27], the shear strain in the two interlayers is given by

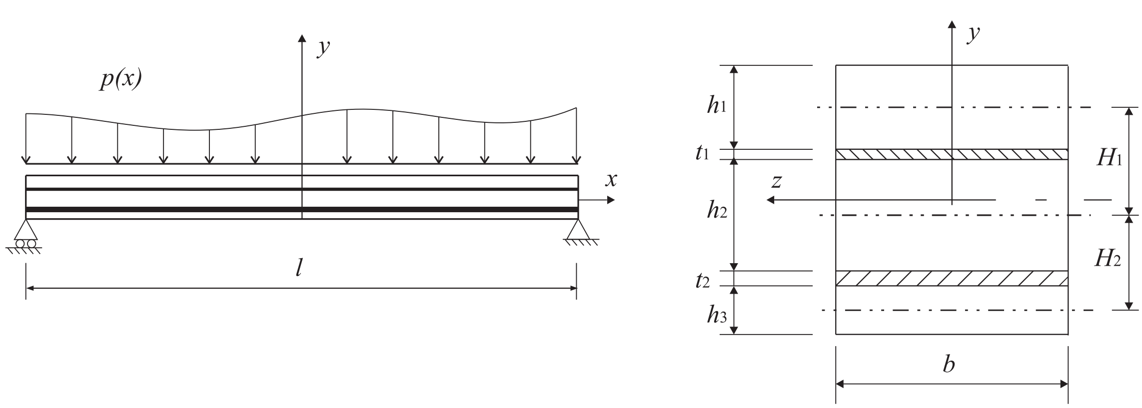

Fig. 1. Five-layered simply-supported laminated glass beam composed of three glass plies bonded by polymeric interlayers. Longitudinal view and cross section (not in the same scale).