Composites: Part B 59 (2014) 123–132

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Nonlinear free vibration, postbuckling and nonlinear static deflection of piezoelectric fiber-reinforced laminated composite beams

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ARTICLE INFO

Article history:
Received 11 September 2013
Received in revised form 28 October 2013
Accepted 14 November 2013
Available online 24 November 2013

Keywords:
A. Laminates
A. Smart materials
B. Vibration
C. Analytical modeling

ABSTRACT

Large static deflection, mechanical and thermal buckling, postbuckling and nonlinear free vibration of laminated composite beams with surface bonded piezoelectric fiber reinforced composite (PFRC) layers under a combined mechanical, thermal and electrical loading are studied in this paper. The temperature rise is considered to be one-dimensional steady state heat conduction in the thickness direction. The governing equations of the piezoelectric fiber reinforced laminated composite beams are derived based on Euler–Bernoulli beam theory and geometric nonlinearity of von Kármán. Rectangular representative volume element (RVE) with rectangular fibers has been considered for piezoelectric fiber reinforced composite. Analytical solution of nonlinear bending and postbuckling analyses has been carried out. A perturbation method is then employed to determine the nonlinear vibration behavior and the nonlinear natural frequencies of the beams with simply supported and clamped boundary conditions. Post-buckling load–deflection and maximum transverse load–deflection relations have been obtained for the beam under consideration. The effects of the temperature rise, beam geometry parameter, and the volume fraction of the piezoelectric fibers on the linear and nonlinear fundamental natural frequencies of the piezoelectric fiber reinforced composites are investigated through a comprehensive parametric study.

1. Introduction

Piezoceramics are commonly brittle and usually used as patched actuators and sensors. Piezoelectric fiber reinforced composite materials have presented as the new class of smart materials. A new piezoelectric fiber reinforced composite (PFRC) developed by Mallik and Ray [1]. The important feature of this PFRC materials is that the monolithic piezoelectric fibers are longitudinally reinforced in the epoxy matrix material. This PFRC material may also be used as sensors and actuators. These materials provide the structures with self-monitoring and self-controlling capabilities when embedded or bonded with in flexible structures. The nonlinear deflection and dynamic analyses of functionally graded material plates with a distributed PFRC actuator were conducted by Ray and Sachade [2], Panda and Ray [3], and Shen [4].

Several theoretical studies have been performed to investigate the response of structures with piezoelectric layers. For instance, Rafiee et al. [5,6] investigated the nonlinear vibration and dynamic behavior of the simply supported piezoelectric functionally graded shells under electrical, thermal, mechanical and aerodynamic loading. Analysis of structures with laminated piezoelectric triangle shell elements has been studied by Tzou and Ye [7]. The nonlinear thermal buckling behavior of carbon nanotube reinforced composite beams with piezoelectric layers is studied by Rafiee et al. [8]. Khedeir et al. [9] obtained an analytical models and solutions for the vibration of laminated beams with extension piezoelectric actuators. Kapuria and Yasin [10] performed the active vibration control of piezoelectric laminated beams with electroded actuators and sensors using finite element involving an electric node. Modeling and analysis of piezoelectric beams using higher order shear deformation theory studied by Elshafei and Alreaiess [11]. Khdeir and Aldraihem [12] obtained the behavior of laminated arches with extension and shear piezoelectric layers. Furthermore, Multi-scale analysis of laminated plates with piezoelectric fiber composite actuators studied by Cook and Vel [13]. Analysis of composite plates with piezoelectric actuators for vibration control using layerwise displacement theory conducted by Han and In [14]. Tan et al. [15] studied the dynamic characteristics of a beam system with active piezoelectric fiber reinforced composite layers. In another study, the dynamics of active piezoelectric damping composites presented by Arafa and Baz [16]. Zhang and Shen [17] conducted the three-dimensional analysis for rectangular 1–3 piezoelectric fiber-reinforced composite laminates with the

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1359-8368/$ - see front matter
http://dx.doi.org/10.1016/j.compositesb.2013.11.017

interdigitated electrodes under electromechanical loadings. Enhancement of pre-buckling behavior of composite beams with geometric imperfections using piezoelectric actuators has been performed by Faria and Almeida [18]. Recently, Youzera et al. [19] studied the nonlinear damping and forced vibration analysis of laminated composite beams. In their analytical formulation, both normal and shear deformations are considered in the core by using the higher-order zig–zag theories.

The main goal of this paper is to further investigate the nonlinear response of the piezoelectric fiber reinforced composite beams in thermal environment. To the best of authors' knowledge, no previous study regarding the nonlinear response of piezoelectric fiber reinforced composite beams has been reported in the open literature. The governing equation of motion of the thermo-piezoelectric buckling is obtained by a series of theoretical derivations and the large deflection and the critical buckling load is received consequently. The Galerkin's method and multiple time scales perturbation scheme are employed to obtain the analytical solution of the nonlinear free vibration problem. Some meaningful results have been obtained.

2. Theoretical modeling

Consider a laminated piezoelectric composite beam with length L and host thickness h containing two piezoelectric fiber reinforced layers of thickness h_p. The total thickness of the composite beam is H., as shown in Fig. 1. The coordinate oxz is set at the midplane of the laminated composite beam. The principal direction of the material is assumed to be parallel to the coordinate axis.

It is assumed that the PFRC layer is made from a mixture of piezoelectric fiber and isotropic matrix. The constituent materials have linear elastic behavior throughout the deformation. The beam is initially free of stress at T_0 (in Kelvin) and is subjected to a uniform temperature rise $\Delta T = T - T_0$.

The effective Young's modulus, shear modulus and Poisson's ratio of PFRC can be expressed by the rule of mixture: [4]

$$E_{11p} = V_F E_{11}^F + V_m E^m \tag{1a}$$

$$\frac{1}{E_{22p}} = \frac{V_F}{E_{22}^F} + \frac{V_m}{E^m} - V_F V_m \frac{v_F^2 E^m/E_{22}^F + v_m^2 E_{22}^F/E^m - 2v^F v^m}{V_F E_{22}^F + V_m E^m} \tag{1b}$$

$$\frac{1}{G_{12p}} = \frac{V_F}{G_{12}^F} + \frac{V_m}{G^m} \tag{1c}$$

where E_{11}^F, E_{22}^F and G_{12}^F are the Young's moduli and shear modulus of piezoelectric fibers, respectively, and E^m and G^m represent the Young's moduli and shear modulus of the isotropic matrix. V_F and V_m refer to the volume fractions of the fibers and the matrix, respectively.

The thermal expansion coefficients in the longitudinal and transverse directions of the beam can be written as

$$\alpha_{11p} = \frac{V_F E_{11}^F \alpha_{11}^F + V_m E^m \alpha^m}{V_F E_{11}^F + V_m E^m} \tag{2a}$$

$$\alpha_{22p} = (1 + v^F) V_F \alpha_{22}^F + (1 + v^m) V_m \alpha^m - v_{12}\alpha_{11} \tag{2b}$$

where α_{11}^F; α_{22}^F and α^m are thermal expansion coefficients, and v_{12}^F and v^m are Poisson's ratios, respectively, of the fibers and matrix. Poisson's ratio and mass density ρ can be obtained by

$$v_{12p} = V_F v^F + V_m v^m, \quad \rho_p = V_F \rho^F + V_m^h \rho^m \tag{3}$$

where v_{12}^F and v^m are Poisson's ratios of fibers and matrix, respectively. The piezoelectric moduli e_{31} and e_{32} can be written by [1]

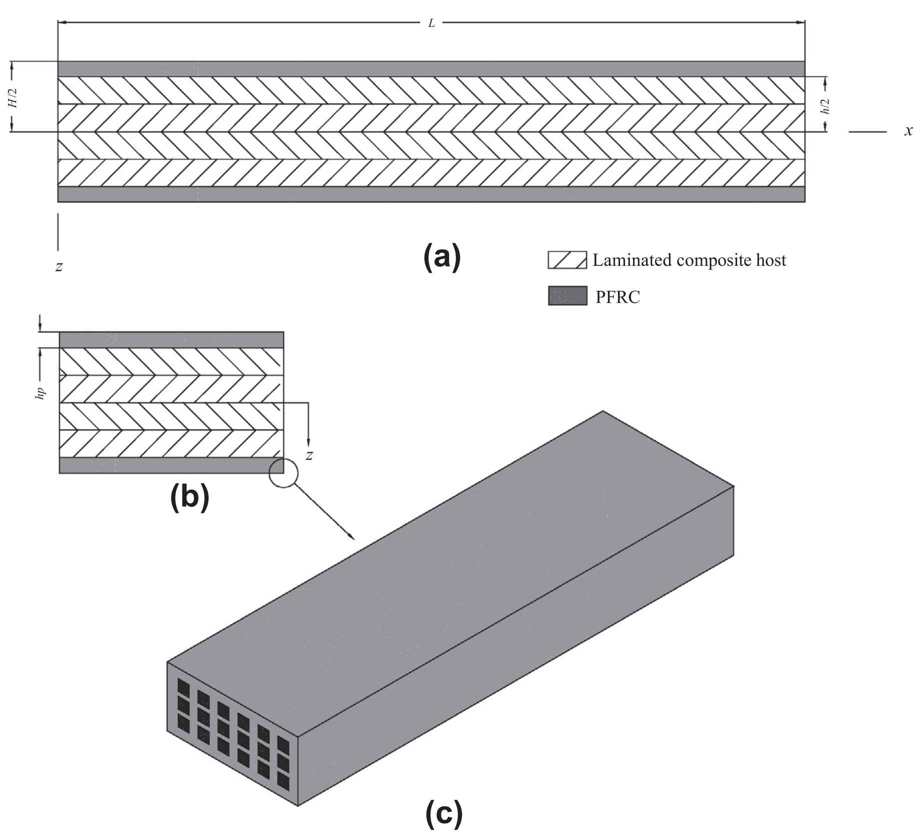

Fig. 1. Configurations of the piezoelectric fiber reinforced composite beam. (a) Front view with material composition and related parameters through the length, (b) Cross section, and (c) Configurations of PFRC lamina.