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Modeling of soft composites under three-dimensional loading

F. López Jiménez *

Laboratoire de Mécanique des Solides, UMR CNRS 7649, École Polytechnique, 91128 Palaiseau Cedex, France

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ABSTRACT

The finite deformation response of fiber-reinforced hyperelastic solids under three-dimensional loading is studied through finite element simulations. The composites are modeled using representative volume elements with random fiber arrangement and periodic boundary conditions. Different matrices and volume fractions are considered. It is found that the shear stiffness of composites with Neo-Hookean components depends on the direction of the applied deformation even when the fibers are not stretched, which indicates a clear dependance on not only the $\bar{I}_1$ and $\bar{I}_4$ invariants, but also on $\bar{I}_5$. This anisotropy increases with the fiber concentration. The effect of using an Ogden matrix with increased nonlinearity is also discussed. Finally, the simulations are compared with suitable homogenization techniques available in the literature. A prediction using two different values of the shear stiffness is able to accurately model the response regardless of the loading direction.

1. Introduction

Soft solids reinforced with significantly stiffer fibers are used in technological applications (car tires, carbon fiber-reinforced elastomers) and can also be found in biological tissue (cornea, arterial walls). The difference in stiffness between the two components can reach several orders of magnitude, which leads to micromechanics not observed in traditional fiber composites, such as significant changes in the fiber orientation.

Phenomenological models for the large deformation of fiber reinforced solids usually follow the framework of Spencer [24], considering a homogeneous solid whose strain energy density $\widetilde{W}$ is the sum of two terms:

$$\widetilde{W}(\bar{I}_1,\bar{I}_2,\bar{I}_3,\bar{I}_4,\bar{I}_5) = \widetilde{W}_{iso}(\bar{I}_1,\bar{I}_2,\bar{I}_3) + \widetilde{W}_{aniso}(\bar{I}_4,\bar{I}_5) \quad (1)$$

where the first term represents an isotropic material, and the second term takes into account the anisotropy introduced by the presence of the fiber reinforcements. The invariants $\bar{I}_1$ to $\bar{I}_3$ are the invariants of the average Cauchy-Green deformation gradient $\bar{\mathbf{C}} = \bar{\mathbf{F}}^{T}\bar{\mathbf{F}}$. These three invariants are isotropic, as opposed to the fourth and fifth: $\bar{I}_4 = \mathbf{N}^{T}\bar{\mathbf{C}}\mathbf{N}$ measures the stretch in the fiber direction, defined by the vector $\mathbf{N}$, while $\bar{I}_5 = \mathbf{N}^{T}\bar{\mathbf{C}}\bar{\mathbf{C}}\mathbf{N}$ depends on the fiber stretch and shear and has no straightforward physical interpretation. In most models each term in Eq. (1) depends on a single invariant, $\widetilde{W}_{iso} = \widetilde{W}_{iso}(\bar{I}_1)$ and $\widetilde{W}_{aniso} = \widetilde{W}_{aniso}(\bar{I}_4)$.

Determining the function $\widetilde{W}$ for matrix-dominated deformations is particularly difficult. It is not equal to the strain energy density of the bulk matrix under the same macroscopic deformation, since it needs to take into account the effect of the inclusions, ranging from stress concentrations to possible changes in microstructure as a result of finite deformations. This is sometimes achieved by fitting experimental results under different loading conditions to the force–displacement relationships obtained from Eq. (1) [20,21]. When the experimental data available is not sufficient, as in the case of fiber reinforced elastomers, models found in the literature often use simple geometrical approximations based on periodic microstructures [5,8].

Estimates for the in-plane response of hyperelastic fiber-reinforced solids with particular microstructures using the second-order homogenization scheme have been presented by Ponte Castañeda and co-workers [16,1]. deBotton [3,4] produced estimates for sequentially-coated composites, obtained by successive lamination of the previous composite with thin layers of the matrix phase. These analytical predictions have been compared with finite element simulations [18] with good agreement up to moderate values of the volume fraction. Lopez-Pamies and Idiart [15] have recently proposed an iterative homogenization technique and used it to produce estimates for the three-dimensional response of fiber reinforced elastomers with a random distribution of parallel fibers. This is particularly interesting to model the in-plane buckling of such composites under bending [11,9], since such instabilities result in complex three-dimensional deformation of the material. The applicability of these predictions to three-dimensional loading has not yet been contrasted numerically or experimentally.

This paper presents a series of numerical simulations in two-dimensional and three-dimensional representative volume elements (RVEs) of fiber-reinforced elastomers, with different fiber

* Tel.: +33 686178830.
E-mail address: lopez@lms.polytechnique.fr

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volume fractions and loading directions. The results are compared to current homogenization models, whose hypothesis and range of validity are discussed. In particular, it will be shown that the shear stiffness of the composite depends on the loading direction, even within the linear regime.

2. Computational model

The composite is idealized as a soft hyperelastic solid reinforced with perfectly parallel cylindrical fibers several orders of magnitude stiffer than the matrix. All the fibers have the same radius r. The fiber volume fraction is V_f, and four values ranging from 0.2 to 0.5 are considered. It is assumed that no voids exist in the composite, and bonding between fibers and matrix is perfect. The strain softening reported by López Jiménez and Pellegrino [11] is therefore neglected here.

A series of 2D and 3D finite element simulations on representative volume elements (RVEs) have been performed using the commercial package Abaqus. Schematics of the geometry and reference systems for the models are shown in Fig. 1. The RVEs are square in the plane perpendicular to the fiber direction, $L_2 = L_3 = L$, and have length L_1 along the fiber direction.

The following subsections give details on the model, such as material properties used for each component, fiber arrangement, size of the RVE and boundary conditions.

2.1. Material properties

The fibers are modeled as a Neo-Hookean material with elastic shear modulus μ_f. The Poisson's ratio is taken as $\nu_f = 0.3$, although the simulations show that the fibers behave as a rigid body, and so the results are insensitive to the value of ν_f. The matrix has been modeled as an incompressible Ogden hyperelastic solid [19], in which the strain energy W_m takes the form:

$$W_m = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) \quad (2)$$

In order to check the effect of the large strain behavior of the matrix, three different combinations of N and α_i have been considered. The first one, $N = 1$ and $\alpha = 2$, corresponds to a Neo-Hookean material. The other two are respectively softer and stiffer at large elongations; the values are shown in Table 1, and the response of all three matrices to uniaxial tension is shown in Fig. 2. The elastic shear modulus is $\mu_m = \sum_{i=1}^{N} \mu_i$. It is assumed that $\mu_m \ll \mu_f$, which is the case of typical elastomers reinforced with materials such as steel or carbon fibers, as well as several biological tissues. In this case, the results can be scaled by μ_m, see Section 4.1.

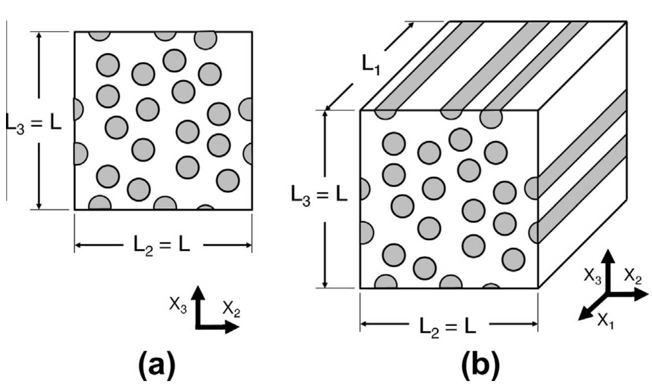

Fig. 1. Representative volume elements (RVEs) used in: (a) two-dimensional and (b) three-dimensional simulations.

Table 1
Parameters of the Ogden hyperelastic energy functions.

	Matrix 1	Matrix 2	Matrix 3
α_1	2	1.2	2.5
μ_1/μ_m	1	0.8	1.01
α_2	0	1	5
μ_2/μ_m	0	0.2	0.02
α_3	0	0	−1
μ_3/μ_m	0	0	−1

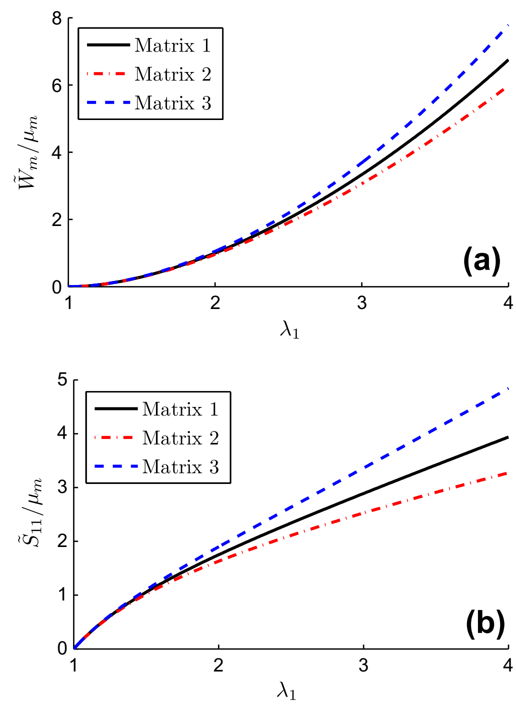

Fig. 2. Response of the three matrices to uniaxial tension: (a) strain energy density normalized by the initial shear stiffness vs. elongation and (b) nominal stress normalized by initial shear stiffness vs. elongation.

2.2. Boundary conditions and applied loading

Periodic boundary conditions are applied in all faces of the RVE using the command `EQUATION` in Abaqus. This requires the mesh to be identical in all opposite faces of the RVE. The conditions can be summarized as:

$$\begin{aligned} \mathbf{u}(L_1, X_2, X_3) - \mathbf{u}(0, X_2, X_3) &= \overset{1}{\mathbf{u}} \\ \mathbf{u}(X_1, L_2, X_3) - \mathbf{u}(X_1, 0, X_3) &= \overset{2}{\mathbf{u}} \\ \mathbf{u}(X_1, X_2, L_3) - \mathbf{u}(X_1, X_2, 0) &= \overset{3}{\mathbf{u}} \end{aligned} \quad (3)$$

where $\overset{i}{u}_j = \overline{F}_{ij} L_j$, L_j is the length of the RVE in the jth direction, and $\overline{\mathbf{F}}$ is the applied deformation gradient, $\overline{F}_{ij} = \partial x_i / \partial X_j$.

The components of $\overset{i}{\mathbf{u}}$ are the displacements of auxiliary dummy nodes, in which displacement or loadings can be imposed. Due to the high difference in stiffness between fibers and matrix, the response of the composite is dominated by the fiber behavior for any deformation involving stretching of the fibers, $\bar{I}_4 \neq 1$. Since the goal is to study the dependance on other invariants, loading will be limited to cases in which no stretching is imposed on the fibers.