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Surface and nonlocal effects on the nonlinear free vibration of non-uniform nanobeams

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ABSTRACT

The surface and nonlocal effects on the nonlinear flexural free vibrations of elastically supported non-uniform cross section nanobeams are studied simultaneously. The formulations are derived based on both Euler–Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT) independently using Hamilton's principle in conjunction with Eringen's nonlocal elasticity theory. Green's strain tensor together with von Kármán assumptions are employed to model the geometrical nonlinearity. The differential quadrature method (DQM) as an efficient and accurate numerical tool in conjunction with a direct iterative method is adopted to obtain the nonlinear vibration frequencies of nanobeams subjected to different boundary conditions. After demonstrating the fast rate of convergence of the method, it is shown that the results are in excellent agreement with the previous studies in the limit cases. The influences of surface free energy, nonlocal parameter, length of non-uniform nanobeams, variation of nanobeam width and elastic medium parameters on the nonlinear free vibrations are investigated.

1. Introduction

Due to superior mechanical, electrical and thermal performances of the nanostructures with respect to the conventional structural materials, they have attracted much attention in modern science and technology in recent years; for example, in micro/nano electromechanical systems (MEMS/NEMS) [1] and biosensors [2]. Hence, accurate prediction of their vibrational behaviors becomes essential for engineering design and manufacture.

Since the controlled experiments on nanoscale are difficult to perform, the mechanical behaviors of the nanostructures are usually investigated using mathematical simulations such as atomistic, atomistic–continuum mechanics and continuum mechanics approaches. On the other hand, the atomistic and atomistic–continuum mechanics simulation methods consume much time and are computationally expensive for analyzing large scale systems. Thus, because of the simplicity and accuracy, continuum mechanic approaches are often used. However, the classical continuum theory cannot predict the size (small scale) effect and size dependence of material properties of nanostructures. The small scale effect and the size dependence of material properties are due to the long-range inter-atomic interaction and the energy associated with atoms at free surfaces of the nanostructures, respectively. These effects are separately simulated based on the continuum approaches.

In order to include the small scale effect, it has been suggested that nonlocal continuum theory developed by Eringen [3–5] could be used in the continuum models for accurate prediction of mechanical behaviors of nanostructures [6]. Nonlocal theory of Eringen is based on this assumption that the stress at a material point is considered as a function of the strain field at all material points in the continuum body. The inter-atomic forces and atomic length scales directly come to the constitutive relations as material parameters [3–5].

In traditional continuum mechanics, the surface free energy is neglected in comparison with the bulk energy because it is associated with only a few layers of atoms near the surface and the ratio of the volume occupied by the surface atoms and the total volume of material of interest is extremely small [7]. As the structural size decreases towards the nanoscale regime, due to the high surface/volume ratio, the surface-to-bulk energy ratio increases. Hence, the surface free energy becomes a significant part of the total elastic potential energy and should be taken into account.

Both of the experimental observations [8] and theoretical analyses [9] indicate that surface layers differ from their bulk counterparts in that their elastic responses are intrinsically size-dependent and consequently, the physical and chemical properties of nanomaterials become size-dependent. Gurtin and Murdoch [10,11] presented a surface elasticity theory by modeling the surface as a two-dimensional membrane adhering to the underlying bulk

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material without slipping to account the effect of surfaces/interfaces on mechanical properties. It has been shown that with correctly choose surface elastic properties, this surface elasticity theory explains various size-dependent phenomena at the nanoscale and the predictions fit well with atomistic simulations and experimental measurements [7,12,13].

In recent years, some studies have been performed to investigate the surface effects on the linear and nonlinear free vibration behaviors of uniform section nanobeams; see for example Refs. [14–21]. However, in these interesting studies the nonlocal effect was not considered. On the other hand, based on the nonlocal constitutive relation of Eringen and without including the surface effects, some researchers attempted to develop nonlocal nanobeam models and applied them to analyze vibration behaviors of nanorods and nanobeams; see for example Refs. [22–30]. Only a few number of studies investigated the surface and nonlocal effects together on the linear free vibration behaviors of nanobeams [31,32]. Lee and Chang [31] obtained the natural frequency of nanotubes using the nonlocal Timoshenko beam theory with consideration of surface effects. Results showed that the nonlocal effect on the frequency ratio is significant, particularly for a smaller value of aspect ratio and at the higher-order modes. In addition, the frequency ratio also increased when the surface effects were taken into account in the analysis. In another work, they examined the surface and small-scale effects on frequency of a non-uniform nanocantilever beam using the nonlocal elasticity theory [32]. Results showed that the frequency of the nanocantilever beam is sensitive to the surface and nonlocal effects.

From the literature survey, it is found that the surface and nonlocal effects on the nonlinear free vibration analysis of nanobeams have been investigated separately. On the other hand, the previous studies showed that both these phenomenon have significant effects on the vibrational behaviors of nanobeams. Consequently, to perform an accurate vibration analysis, the formulation should include both these effects. In most practical circumstances, however, thin elastic nanobeams commonly sustain large deformation where the deflection is of order of the nanobeams thickness. The infinitesimal deformation model then is invalid, and a geometrically nonlinear model is evidently needed. Motivated by this consideration, this paper investigates the nonlinear free vibration of variable width nanobeams embedded in an elastic medium with consideration of surface and nonlocal effects simultaneously. The formulation is derived based on both Euler–Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT) in conjunction with von Kármán geometric nonlinearity. The differential quadrature method (DQM) is employed to solve the nonlinear free vibration governing equations of nanobeams with arbitrary boundary conditions. The effects of surface elasticity, residual surface tensions, nonlocal parameter, variation of nanobeam width, elastic medium parameter and transverse shear deformation of the nanobeams on the nonlinear natural frequencies of the nanobeams are studied.

2. Mathematical modeling

Consider a nanobeam of length L, varying width $b(x)$ and thickness h as shown in Fig 1. A Cartesian coordinate system (x,z) is used to label the material points of the nanobeam in the unstressed reference configuration. The displacements $\bar{u}$ (in the x-direction), $\bar{w}$ (in the z-direction) can be approximated as,

$$\bar{u}(x,z,t) = u(x,t) - z\frac{\partial w}{\partial x} + z\lambda\left(\frac{\partial w}{\partial x} + \varphi\right), \quad \bar{w}(x,z,t) = w(x,t) \tag{1a, b}$$

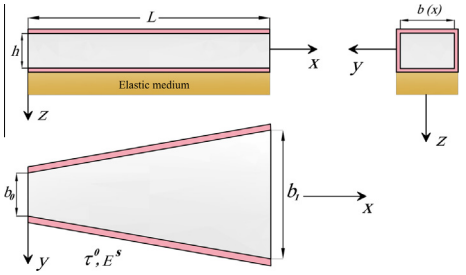

Fig. 1. The geometry of nanobeam.

where u and w are the axial and transverse displacement components of a material point on the mid-plane of the beam (i.e. $z = 0$); φ is the bending rotation of beam cross section about the y-axis; also, λ is a constant parameter, which switch Eq. (1a) to EBT $\lambda = 0$ or TBT $\lambda = 1$.

Using Eq. (1), the nonzero Green's strain tensor components subjected to von Kármán assumptions in terms of displacement and rotation components become,

$$\varepsilon_{xx} = \varepsilon^0_{xx} - z\frac{\partial^2 w}{\partial x^2} + z\lambda\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x}\right),$$
$$\varepsilon^0_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \gamma_{xz} = \lambda\left(\frac{\partial w}{\partial x} + \varphi\right) \tag{2a–c}$$

The classical constitutive relation of the surface boundaries ($y = \pm b/2, z = \pm h/2$) as given by Gurtin and Murdoch [10,11] and also the classical constitutive relations for the internal material of the beam ($-b/2 < y < b/2$, $-h/2 < z < h/2$) can be expressed as

$$\sigma^s = \tau^0 + E^s\varepsilon_{xx}, \quad \sigma_{xx} = E\varepsilon_{xx}, \quad \tau_{xz} = \lambda k_s G\left(\frac{\partial w}{\partial x} + \varphi\right) \tag{3a–c}$$

where τ^0 and E^s are the residual surface tension in the axial direction and the surface elastic modulus, respectively; also, E, G and k_s are Young's modulus, shear modulus and shear correction factor of the internal material of the beam. The stress resultants, in general are defined as

$$N_{xx} = \int_{-h/2}^{h/2}\sigma_{xx}b\,dz + \oint_\Gamma \sigma^s ds = (EA)^*\varepsilon^0_{xx} + 2\tau^0(b+h), \quad Q_x = \int_{-h/2}^{h/2}\tau_{xz}b\,dz = \lambda k_s GA\left(\frac{\partial w}{\partial x} + \varphi\right)$$
$$M_{xx} = \int_{-h/2}^{h/2} z\sigma_{xx}b\,dz + \oint_\Gamma z\sigma^s ds = (EI)^*\left[-\frac{\partial^2 w}{\partial x^2} + \lambda\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x}\right)\right] \tag{4a – c}$$

where $A = bh$ and the effective in-plane and flexural rigidities are,

$$(EA)^* = EA + 2E^s(b+h), (EI)^* = E\left(\frac{bh^3}{12}\right) + E^s\left(\frac{h^3}{6} + \frac{bh^2}{2}\right) \tag{5a, b}$$

Based on Eringen's nonlocal elasticity theory [3–5], size effects are taken into account by the integration of a scale parameter into classical continuum models. In the nonlocal elasticity theory, the stress at a reference point is assumed to be a functional of the strain field at every point in the body. In this way, the internal characteristic length can be considered in the constitutive relations simply as a material parameter by the following differential constitutive relation

$$\left(1 - \mu\frac{\partial^2}{\partial x^2}\right)\sigma^{nl}_{ij} = \sigma^l_{ij}, \quad \mu = (e_0 l)^2 \tag{6a, b}$$