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Optimum laminate design by using singular value decomposition

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ABSTRACT

In practice, a structure is subjected to given loads and boundary conditions, and a multitude of stress and strain states may exist in the structure; hence, optimal construction of a laminate in a structure cannot be sought by considering only a limited number of stress resultants in the existence of multiple load cases. Then, another design objective based on optimization of a laminate for the worst possible load case emerges which is formulated as a minimax problem whose solution is shown to be equivalent to singular value minimization problem. As the squares of singular values are the bounds of power, energy and power spectral density ratios between the input and output vectors, shaping the singular values of a composite material is equivalent to shaping the response of the material. As a novel approach, singular values are used for the layout optimization of laminate. In this method, the main idea is minimization of the largest singular value of the transfer function matrix between force/moment resultants and outputs stress/strain. Thus the overall optimization problem is reduced to a simple minimization problem. Numerical examples and finite element simulations are presented for several test problems. In particular, it is shown that the use of singular values and singular vectors is computationally advantageous in case of multiple load case.

1. Introduction

Mechanics of composite materials and design aspects of these materials have been the subject of numerous studies in literature in which material stiffness or strength is optimized by using some numerical or graphical techniques [1–3]. While continuous optimization techniques were used for solving optimization problems, researchers recently have focused on discrete optimization techniques since design of a composite laminate stacking sequence employs discrete optimization techniques to determine discrete layer thickness values and orientation angels. In general, composite materials are designed for minimum weight, optimized stiffness or maximum laminate strength. Since minimum weight design formulations do not depend on the orientation of the layers, convergence difficulties may arise in optimization tasks; therefore, formulations based on maximization of laminate strength are proposed [4,5]. For instance, for some in-plane load cases, a quadratic first-play failure criterion based on an approximate failure envelope in the strain space is used in [5]. In order to overcome this difficulty, Huang and Kroplin [6] used a two-level design process for optimum laminate design by successively optimizing the ply orientation angles to minimize the strain energy and ply thickness to minimize the weight. Zhang et al. proposed an extended stress-based method for orientation angle optimization of laminated composite structures [7]. An integrated model for optimum weight design of symmetrically laminated composite plates subjected to dynamic excitation is presented in [8]. A new variant of the simulated annealing algorithm was proposed to optimize the lay-up design in [9] where they aimed to minimize the thickness (or weight) of laminated composite plates subject to both in-plane and out-of-plane loading. Irisarri et al. [10] presented a multiobjective optimization methodology for composite stiffened panels. The work of Kim et al. [11] can also be cited where by using an optimal design formulation based on the state space method, Tsai-Wu failure criterion is used as the objective function for optimal laminate design. In [12], stacking sequence combination of a laminate is optimized such that the strain under an applied load is reduced with the lowest ply number. The loads are considered to be varying over the large composite panels and a global optimization algorithm is used to obtain the optimum design in [13]. The works [14] for minimum weight design of panels for post buckling response, [15] for optimal design of laminates for fundamental frequency, [16] for optimum design of shells for maximizing the energy-absorption, [17] for optimum design of tapered laminates, and [18] for optimal design of composite turbine blades can also be cited. Zhang et al. [19], proposed an extended stress-based method to reduce the huge number of the design variables in the orientation optimization of composite structures. They minimized the mean compliance under the multiple load cases or maximized the eigenvalues of a composite structure. The design of the composite laminated lay-ups was formulated as discrete multi-material selection problems [20]. The minimum compliance, mass constrained multiple load case problem was formulated and solved in their study. Shokrieh et al. [21], were concerned in their study with

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designing modified tabbing systems for the testing of composite thin-walled tubes with a symmetric layup. In their study, stiffness was maximized at minimum weight by developing a minimum weight optimization method for sandwich structure under combined torsion and bending loads. Stegmann and Lund have proposed the Discrete Material Optimization (DMO). The method was used for material optimization of general composite laminate shell structures. The method uses gradient information combined with mathematical programming to solve a discrete optimization problem. The DMO method is derived from multi-phase material optimization in the sense that element stiffness is computed from a weighted sum of candidate materials. The aim of the optimization is for each layer to choose the material from the set of candidate materials either isotropic or orthotropic with a given fiber angle [22]. Later Lund and Stegmann [23], used this method (DMO) on multi-phase topology optimization where the material stiffness was computed as a weighted sum of candidate material and applied the method to wind turbine blades. Bruynell and Fleury [24] used sequential convex programming for optimization of composite structures. This is an approximation concept to solve the optimization problem. Bruyneel and Duysinx published a note on the design of laminates subject to restrictions on the ply strength. The minimum weight design was considered. It was shown that the formulation includes singular optima, similar to the ones observed in topology optimization including local stress constraints [25]. Bruyneel [26], in his paper proposed a new parameterization of the mechanical properties for the optimal selection of materials. He compared his approach with multi-phase topology optimization (i.e. Discrete Material Optimization—DMO). Bruyneel computed material stiffness as a weighted sum of the candidate material properties, and the weights based on the shape functions of a quadrangular first order finite element. Found out that, compared to DMO, this method (SFP) required fewer design variables. Bruyneel et al. [27] developed an optimization procedure based on multi-phase topology optimization. Their formulation relied on the SFP (Shape Functions with Penalization) parameterization, in which the discrete optimization problem is replaced by a continuous approach with a penalty to exclude the intermediate values of the design variables. Gao et al. [28] proposed a parameterization scheme named bi-value coding parameterization (BCP). This method also used to deal with the layout design of the discrete material orientations for laminate composite. The theoretical expression of the BCP was constructed in an explicit way for any number of materials. The BCP can be considered as an extension of SFP and particularly well suited for optimization problems with a huge number of discrete orientation or candidate materials. However DMO/SFP/BCP methods are presented to find structural topology and optimal fiber orientations.

In most of the studies cited above, it is common that laminate parameters are optimized for certain load cases and some averages of stress values, constant stress resultants, moment resultants and/or shear resultants are employed in designing the laminates; hence, such designs are only optimal for some load cases for which they are optimized. In practice, a structure is subjected to given loads and boundary conditions and a multitude of stress and strain states may exist in a structure; hence, optimal construction of a laminate in a structure cannot be sought by considering only a limited number of stress resultants in the existence of multiple load cases. Therefore, another design objective, called maximization of stiffness or minimization stress for the worst possible load case, emerges to take into account all possible load cases. Motivated by this fact in this paper, optimum laminate design problem for the worst possible load case is formulated as a minimax problem whose solution is shown to be equivalent to a singular value minimization problem; subsequently, singular values are employed for designing laminates.

The singular value decomposition (SVD) is a very powerful tool that is utilized for studying input–output properties in multivariable control systems, e.g., [29,30]. The SVD is employed for studying the effects of mode localization on the input–output directional properties of structures in [31] and analyzing design sensitivities of structures in [32]. Gerzen et al. [33] studied the inner structure of sensitivities in nodal based shape optimization by the singular value decomposition. Furukawa and Michopoulos presented a methodology that updates the loading path at every sensor reading to identify elastic moduli of anisotropic materials; they applied the SVD to the reliability of elastic moduli [34]. In this paper, the SVD is applied to continuous formulations of optimum laminate design problem. Considering stress and strain equations, laminated composite materials are optimized for the worst possible load case by using singular values. Singular values of the equations of a structure have a special meaning since the squares of singular values are bounds of power, energy and power spectral density ratios between the input and output vectors; thus, shaping the singular values of a structure is equivalent to shaping the behavior of the structure [35]. The SVD based analysis is well suited to study input–output directional relationships, because associated singular vectors tell us how the outputs (strain or stresses) are related to the inputs (loads). Numerical examples are presented to illustrate the proposed approach. Briefly, it is shown that using singular values is computationally advantageous in case of multiple load case.

The outline of the article is as follows; Some properties of the SVD are revised in Section 2, strength and stiffness design problems of laminates are summarized in Section 3. and then the SVD is introduced to laminate equations in Section 4. The proposed approach is implemented into model problems in Section 5, and conclusions are drawn in Section 6.

2. Properties of the singular value decomposition

Note that the exposition of the material on the SVD is based on that of [36–38]. Consider the matrix $\mathbf{A} \in C^{m\times n}$, then there exist unitary matrices $\mathbf{U} \in C^{m\times m}, \mathbf{S} \in R^{m\times n}$ and $\mathbf{V} \in C^{n\times n}$ called the SVD of $\mathbf{A}$ such that $\mathbf{A}$ can be factored as

$$A = \mathbf{USV}^H \tag{1}$$

where columns of $\mathbf{U} = [\mathbf{u}_1|\mathbf{u}_2||\mathbf{u}_m]$ and $\mathbf{V} = [\mathbf{v}_1|\mathbf{v}_2||\mathbf{v}_n]$ are respectively, the left and right singular vectors, and $\mathbf{V}^H$ is the conjugate transpose of $\mathbf{V}$. If $m = n$, then $\mathbf{S} = Diag\{\mu_1, \mu_2, \ldots, \mu_m\}$; on the other hand, if $m > n$ then,

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_d \\ O_{(m-n)\times n} \end{bmatrix} \tag{2}$$

If $m < n$, then

$$\mathbf{S} = [\mathbf{S}_d \, O_{m\times(n-m)}] \tag{3}$$

where $\mathbf{S}_d = Diag\{\mu_1, \mu_2, \ldots, \mu_m\}$, $p = min(m,n)$, $\mathbf{O}_{i\times j} \in R^{i\times j}$ whose elements are all zero, and μ_i are singular values of $\mathbf{A}$. We adopt the ordering $\mu_1 \geqslant \mu_2 \geqslant \ldots \geqslant \mu_n \geqslant 0$. Note that $\mathbf{u}_i$ and $\mathbf{v}_i$ are respectively, orthonormal eigenvectors of $\mathbf{AA}^H$ and $\mathbf{A}^H\mathbf{A}$; namely,

$$\mathbf{UU}^H = \mathbf{I} \quad \text{and} \quad \mathbf{AA}^H\mathbf{U} = \mathbf{US}^2 \tag{4}$$

$$\mathbf{VV}^H = \mathbf{I} \quad \text{and} \quad \mathbf{A}^H\mathbf{AV} = \mathbf{VS}^2 \tag{5}$$

where $\mathbf{I}$ is the identity matrix. In addition, for a square matrix $\mathbf{A}$

$$\text{if } \mathbf{A} = \mathbf{USV}^H, \; \mathbf{A}^{-1} = \mathbf{VS}^{-1}\mathbf{U}^H \tag{6}$$

Although the singular values of $\mathbf{A}$ are uniquely defined, the singular vectors are not. If $\mathbf{A} = \mathbf{USV}^H$, then $\mathbf{A} = \mathbf{U}'\mathbf{S}\mathbf{V}'^H$, where $\mathbf{U}' = \mathbf{U}e^{j\theta}$, $\mathbf{V}' = \mathbf{V}e^{-j\theta}$ and j is the imaginary unit, is also the SVD of $\mathbf{A}$ for any θ.