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An analytical study on the nonlinear free vibration of functionally graded nanobeams incorporating surface effects

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ABSTRACT

Nonlinear free vibration of simply supported FG nanoscale beams with considering surface effects (surface elasticity, tension and density) and balance condition between the FG nanobeam bulk and its surfaces is investigated in this paper. The non-classical beam model is developed within the framework of Euler–Bernoulli beam theory including the von Kármán geometric nonlinearity. The component of the bulk stress, σ_{zz}, is assumed to vary cubically through the nanobeam thickness and satisfies the balance conditions between the FG nanobeam bulk and its surfaces. Accordingly, surface density is introduced into the governing equation of the nonlinear free vibration of FG nanobeams. The multiple scales method is employed as an analytical solution for the nonlinear governing equation to obtain the nonlinear natural frequencies of FG nanbeams. Several comparison studies are carried out to demonstrate the effect of considering the balance conditions on free nonlinear vibration of FG nanobeams. Lastly, the influences of the FG nanobeam length, volume fraction index, amplitude ratio, mode number and thickness ratio on the normalized nonlinear natural frequencies of the FG nanobeams are discussed in detail.

1. Introduction

Functionally graded materials (FGMs) are classified as novel composite materials. These materials are heterogeneous composite materials, in which the material properties vary continuously from one interface to the other. This is achieved by gradually varying the volume fraction of the constituent materials. During the past decade, FGMs have been widely used in various aspects of engineering sciences, such as aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical, and shipbuilding industries. With the development of the material technology, FGMs have been employed in micro/nano-electro-mechanical system (MEMS/NEMS) [1–3]. Because of high sensitivity of MEMS/NEMS to external stimulations, understanding mechanical properties and behavior of them are of significant importance to the design and manufacture of FG MEMS/NEMS.

Structures at nanometer length scale are known to exhibit size-dependent behavior [4–6]. This is attributed to the fact that the fraction of energy stored in the surfaces becomes comparable to that in the bulk due to the relatively high surface area to volume ratio [7]. The classical continuum theories, in which the surface energy effects are normally neglected, need to be modified to incorporate the size-dependent behavior. It is assumed that the surface is at least a few atomistic layers thick to justify the applicability of the notion of stresses and strains.

Gurtin and Murdoch [8,9] presented a 3-D theory based on continuum mechanics concept that takes into consideration the effects of surface energy. In their work, a surface is regarded as a mathematical deformable membrane of zero thickness fully adhered to the underlying bulk material. The equilibrium and constitutive equations for the bulk are the same as those in the classical theory of elasticity. In addition, a set of constitutive equations and the generalized Young–Laplace equation are applied to the surface. Lim and He [10] developed a model based on the Gurtin–Murdoch theory to analyze the deformations of a nanoscale film under bending. Lu et al. [11] generalized the thin plate model to include the normal stresses in the bulk and presented a modified theory for thin and thick plates. In this regard, they assumed a linear variation through the thickness of the transverse normal stress so that the surface balance equations are satisfied. They presented solutions for deflections (static) and natural frequencies of an infinitely wide plate with finite length. Several studies have fabricated nanometer scale electromechanical beam resonators and examined their responses experimentally [12–15].

In recent years, studies on FG nanoplates and nanobeams with surface effects, as the based NEMS devices, have attracted increasing research efforts. Lü et al. [16,17] developed a generalized refined theory including surface effects for functionally graded

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films. They modeled the film bulk based on the classical Kirchhoff and the classical generalized shear deformable theories. For satisfying the surface balance equations, unlike the linear assumption made by Lu et al. [11] for homogeneous films, the transverse normal stress was assumed to vary in a cubic relation through thickness for FGM films. In another work, Lü et al. [18] investigated nonlinear responses of FGM nano-films incorporating surface effects. They modeled the film bulk based on the Kirchhoff plate theory and considered a linear variation for the transverse normal stress. It was showed that the deflection of nano-film was scaling dependent. Ke et al. [19] investigated the nonlinear free vibration of functionally graded nanocomposite beams reinforced by single-walled carbon nanotubes (SWCNTs) based on Timoshenko beam theory and von Kármán geometric nonlinearity. Ansari et al. [20] considered the free vibration characteristics of microbeams made of functionally graded materials (FGMs) based on the strain gradient Timoshenko beam theory. They showed that the value of gradient index plays an important role in the vibrational response of the FG microbeams of lower slenderness ratios and by increasing the length to thickness ratio of the FG microbeam, the value of dimensionless natural frequency tends to decrease for all amounts of the gradient index. Ke et al. [21] studied nonlinear free vibration of FG microbeams based on the modified couple stress theory and von Kármán geometric nonlinearity. It was found that both the linear and nonlinear frequencies increase significantly when the thickness of the FGM microbeam was comparable to the material length scale parameter. Asgharifard Sharabiani and Haeri Yazdi [22] studied surface effects including surface elasticity and surface tension on nonlinear free vibration of FG nanobeams based on the Euler–Bernoulli beam theory. They did not consider the surface equilibrium condition in derivation of the governing equation.

The main goal of this work is to study the surface effects, including surface elasticity, tension and density, on the nonlinear free vibration of FG nanobeams based on the Euler–Bernoulli beam theory with considering the surface equilibrium condition. The von Kármán geometric nonlinearity is taken into account with the assumption of cubic variation of normal stress through the thickness. The method of multiple scales has been used as an analytical solution for the nonlinear governing equation.

2. Problem formulation

Consider a FG nanobeam with length L ($0 \leqslant x \leqslant L$), thickness h ($-h/2 \leqslant z \leqslant h/2$), and width b ($-b/2 \leqslant y \leqslant b/2$). The FG nanobeam is generally composed of two different materials at the top and the bottom surfaces (as shown in Fig. 1). Poisson's ratio ν is assumed to be constant, i.e. $\nu = 0.3$, whereas bulk elastic modulus $E(z)$, mass density $\rho(z)$, surface elastic modulus $E^s(z)$, and residual surface stress $\tau^0(z)$ are assumed to vary in the thickness direction according to power law distribution:

$$E(z) = (E_1 - E_2)\left(\frac{2z+h}{2h}\right)^m + E_2 \tag{1.a}$$

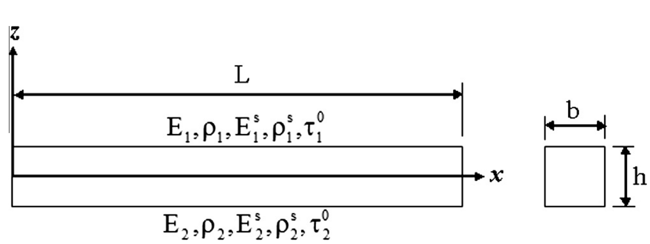

Fig. 1. Geometry of a FG nanobeam.

$$\rho(z) = (\rho_1 - \rho_2)\left(\frac{2z+h}{2h}\right)^m + \rho_2 \tag{1.b}$$

$$E^s(z) = (E_1^s - E_2^s)\left(\frac{2z+h}{2h}\right)^m + E_2^s \tag{1.c}$$

$$\tau^0(z) = (\tau_1^0 - \tau_2^0)\left(\frac{2z+h}{2h}\right)^m + \tau_2^0 \tag{1.d}$$

where, the subscripts 1 and 2 denote the top surface and bottom surface, respectively, and a volume fraction index m determines the variation profile of material properties across the FG nanobeam thickness.

Since there is no slipping between two surface layers and the underlying material, the displacement in the whole beam is continuous. Upon the Euler–Bernoulli beam model, the displacement field at any point of the beam (the bulk and the surface layers) can be written as

$$u_x(x,z,t) = U(x,t) - z\frac{\partial W}{\partial x} \tag{2.a}$$

$$u_z(x,z,t) = W(x,t) \tag{2.b}$$

where t is time, $U(x, t)$ and $W(x, t)$ are displacement components of the mid-plane along x and z directions, respectively.

The von kármán nonlinear strain–displacement relationship is presented as

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} + \frac{1}{2}\left(\frac{\partial u_z}{\partial x}\right)^2 = \frac{\partial U}{\partial x} - z\frac{\partial^2 W}{\partial x^2} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \tag{3}$$

Assuming a FG material and neglecting any residual stresses in the bulk due to surface stress, the relevant bulk stress–strain relation of FG nanobeam can be written as

$$\sigma_{xx} = E(z)\varepsilon_{xx} + \nu\sigma_{zz} \tag{4}$$

The surface constitutive equation [8,23] can be given by

$$(\tau_{xx})_{1,2} = (\tau^0)_{1,2} + (E^s)_{1,2}\left(\frac{\partial U}{\partial x} - z\frac{\partial^2 W}{\partial x^2} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2\right) \tag{5.a}$$

$$(\tau_{zx})_{1,2} = (\tau^0)_{1,2}\left(\frac{\partial W}{\partial x}\right) \tag{5.b}$$

In which τ_{xx} and τ_{zx} are surface stresses.

The stresses of the surface layers must satisfy the following equilibrium relations [8,23]

$$(\tau^+_{\beta i,\beta})_1 - (\sigma^+_{iz})_1 = (\rho^0)_1\left(\frac{\partial^2 u_i^+}{\partial t^2}\right)_1 \quad \text{at } z = +h/2, \tag{6.a}$$

$$(\tau^-_{\beta i,\beta})_2 - (\sigma^-_{iz})_2 = (\rho^0)_2\left(\frac{\partial^2 u_i^-}{\partial t^2}\right)_2 \quad \text{at } z = -h/2, \tag{6.b}$$

where the surface stresses of the FG nanobeam are denoted by $\tau^+_{\beta i}$ and $\tau^-_{\beta i}$. $\sigma^+_{iz} = \sigma_{iz}(z = +h/2)$ and $\sigma^-_{iz} = \sigma_{iz}(z = -h/2)$ are bulk stresses. $u_i^+ = u_i(z = +h/2)$ and $u_i^- = u_i(z = -h/2)$ are the displacement of surface layers in the i-direction at $z = \pm h/2$, respectively. In Eq. (6.a), (6.b), $\beta = x, y$ and $i = x, y, z$.

By substituting Eqs. (2.a), (2.b) and (5.a), (5.b) into (6.a), (6.b), the following equations can be obtained,

$$(\sigma_{zz})_1 = \tau_1^0\frac{\partial^2 W}{\partial x^2} - \rho_1^0\frac{\partial^2 W}{\partial t^2} \tag{7.a}$$

$$(\sigma_{zz})_2 = -\tau_2^0\frac{\partial^2 W}{\partial x^2} + \rho_2^0\frac{\partial^2 W}{\partial t^2} \tag{7.b}$$