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A simple anisotropic hyperelastic constitutive model for textile fabrics with application to forming simulation

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ABSTRACT

A simple hyperelastic constitutive model is developed to characterize the anisotropic and large deformation behavior of textile fabrics. In the model, the strain energy function is decomposed into two parts representing fiber stretches and fiber–fiber interaction (cross-over shearing) between weft and warp yarns. The proposed constitutive model is demonstrated on a balanced plain woven fabric. The actual forms of the strain energy functions are determined by fitting uni-axial tensile and picture-frame shear tests of the woven fabrics. The developed model is validated by comparing numerical results with experimental bias extension data, and then applied to simulation of a benchmark double dome forming, demonstrating that the proposed anisotropic hyperelastic constitutive model is highly suitable to predict the large deformation behavior of textile fabrics.

1. Introduction

Textile composites are of increasing interest in automotive and aerospace engineering due to their significant material properties, such as high formability, high specific strength and specific stiffness as well as improved resistance to impact. Composites industry is strongly motivated to develop adequate design tools for textile composites. In composite sheet forming process, the constitutive behavior of textile composites may be affected by many factors, such as shear angle change, strain distribution and manufacturing parameters [1,2]. One of the main challenges is the development of appropriate constitutive models that can accurately predict the anisotropic behavior of textile composites resulting from complex reorientation and redistribution of yarn fibers [3].

Despite enormous research in the field, there is no widely accepted model that describes accurately all the main aspects of fabric mechanical behavior [4]. Due to the multiscale nature of textile composites, it allows both discrete as well as continuous approaches. In discrete approaches, the mechanical analysis is carried out at unit cell level, in which each yarn in the fabric is modeled [5–14]. Micro-mechanisms such as fiber bending/unbending and contact between fiber yarns inherent in fabric deformation can be investigated and characterized by using the unit cell approach. Hybrid elements, e.g., a combination of beam and membrane elements, have been used to represent both the bending stiffness of yarns and the in-plane properties of fabric [15].

Homogenization method is another approach for material characterization of textile composites [5,7]. Nevertheless, because of very large number of yarns or fibers, the computational effort in these approaches is so significant that the unit cell approaches are limited to small domain analysis and are not applicable to forming analysis and processing optimization.

Conversely to these discrete approaches is to consider textile fabrics as continuum [16–23]. The main benefit of this approach is that it can be used in standard finite element method. Peng and Cao [18] developed a non-orthogonal constitutive model to represent the anisotropic material behavior of woven composites under large deformation. They considered two yarn directions to define the non-orthogonal frame. Initial fiber orientation and the deformation gradient are used to track the reorientation of the fibers during deformation. Using the same frame work of Peng and Cao [18], and Lee et al. [19] modified the non-orthogonal constitutive model by introducing tension coupled shear modulus in the material stiffness matrix. The model was then validated by comparing numerical results with experimental data of shear tests under uniaxial and biaxial tensile loading, showing good agreement with meso-scale analysis results.

A number of researchers have developed hyperelastic constitutive models for characterizing the behavior of textile composite

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fabrics at large strain ([10,20–24]). Khan et al. [22] extended the hypoelastic approach to macro-scale to perform forming simulation of woven fabrics using warp and weft fibers rotation tensor. Their simulation results showed good accordance with experimental data of double dome forming. Aimene et al. [23] proposed a hyperelastic model for composite forming simulation by considering stretching and shearing of fiber yarns.

Based on the fabric material characterization approach proposed by Aimene et al. [23], this paper aims to develop a simple hyperelastic constitutive model to characterize the anisotropic nonlinear material behavior of dry fabrics under large deformation. The strain energy function for the model is decomposed into two parts representing fiber stretches and fiber–fiber interaction (cross-over shearing) between weft and warp yarns. By decoupling the fiber extension energy with in-plane shearing energy, the material characterization of textile fabric is greatly facilitated. The material parameters can be conveniently obtained from two simple tests, namely, uni-axial tensile and shearing tests. The proposed constitutive model is demonstrated on a balanced plain weave fabric.

The structure of the remainder of this paper is given as follows. In Section 2, a simple hyperelastic constitutive model for characterizing large deformation behavior of textile fabric is provided. Experimental uni-axial tensile and picture frame tests are introduced in this section to determine the equivalent material properties of a plain weave composite fabric. The model is validated by comparing numerical results with experimental bias extension data in Section 3 and then applied to simulate a benchmark double dome forming in Section 4. Section 5 gives a summary.

2. A simple hyperelastic model for fabrics

Textile fabrics can be draped as a preform for following-up RTM process. During forming, portion of the energy exerted on textile fabric will be dissipated due to friction between fiber yarns. Nevertheless, in hyperelastic approaches, the energy required to deform textile fabrics can be approximately treated as the strain energy of the corresponding deformations during forming unless the unloading process is considered. The similar treatment is implied in the development of hypoelastic models for woven composite fabrics [10,16,18].

During forming of dry fabric, the main deformation modes are extension along fiber yarns and angular variation between weft and warp yarns. Accordingly, following the hyperelastic constitutive framework proposed by Aimene et al. [23], the strain energy function for dry 2D woven fabrics can be decomposed into two parts representing fiber stretches and fiber–fiber interaction (cross-over shearing) between weft and warp yarns. Assume that the original fiber directional unit vectors for the weft and warp yarns are $\mathbf{a}_0$ and $\mathbf{b}_0$, respectively, then the strain energy due to fiber stretches can be simply characterized by fiber stretch ratios, i.e.,

$$W^F = W^F(\mathbf{C}, \mathbf{a}_0, \mathbf{b}_0) = W^F_a(I^a_4) + W^F_b(I^b_4) \tag{1}$$

where $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ is the right Cauchy–Green deformation tensor. $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$ is the deformation gradient tensor. $\mathbf{X}$ represents the position of a material particle in the original (undeformed) configuration, while $\mathbf{x}$ is the position of the corresponding particle in the current (deformed) configuration. W^F_a and W^F_b represent the weft and warp yarn fiber stretch energies, respectively. I^a_4 and I^b_4 are defined as,

$$I^a_4 = \mathbf{a}_0 \cdot \mathbf{C} \cdot \mathbf{a}_0 = (\lambda_a)^2, \quad I^b_4 = \mathbf{b}_0 \cdot \mathbf{C} \cdot \mathbf{b}_0 = (\lambda_b)^2 \tag{2}$$

where λ_a and λ_b are the stretch ratios of the weft and warp fiber yarns, respectively.

The fiber–fiber interaction energy W^{FF}_{ab} can be represented by the shearing angle between the two families of fibers. As shown in Fig. 1, the original angle between the weft and warp yarns $\mathbf{a}$ and $\mathbf{b}$ is

$$\cos\varphi_0 = \mathbf{a}_0 \cdot \mathbf{b}_0 \tag{3}$$

After deformation $\mathbf{F}$, the angle between the two fiber yarns becomes to φ

$$\cos\varphi = \mathbf{a} \cdot \mathbf{b}/(\|\mathbf{a}\| \cdot \|\mathbf{b}\|) = (I^a_4 I^b_4)^{-\frac{1}{2}}\mathbf{a}_0 \cdot \mathbf{C} \cdot \mathbf{b}_0 \tag{4}$$

Introducing I^{ab}_7 to represent the angle change between the two fiber yarns:

$$I^{ab}_7 = \Delta\varphi = \varphi - \varphi_0 \approx (I^a_4 I^b_4)^{-\frac{1}{2}}\mathbf{a}_0 \cdot \mathbf{C} \cdot \mathbf{b}_0 - \mathbf{a}_0 \cdot \mathbf{b}_0 \tag{5}$$

The fiber–fiber interaction energy can be quantitatively represented by I^{ab}_7, i.e.,

$$W^{FF}_{ab} = W^{FF}_{ab}(I^{ab}_7) \tag{6}$$

In summary, the strain energy function for a dry fabric with two fiber yarns can be decomposed as,

$$W = W(\mathbf{C}, \mathbf{a}_0, \mathbf{b}_0) = W^F_a(I^a_4) + W^F_b(I^b_4) + W^{FF}_{ab}(I^{ab}_7) \tag{7}$$

where W^F is the contribution from fiber stretches, and W^{FF} is the shearing energy resulting from fiber–fiber interaction. The second Piola–Kirchhoff stress tensor is $\mathbf{S} = 2\partial W/\partial\mathbf{C}$. The Cauchy stress tensor, $\boldsymbol{\sigma}$, is given by $\sigma = J^{-1}\mathbf{FSF}^T$,

$$\sigma_{ij} = I_3^{-\frac{1}{2}} F_{ik} F_{jl} \sum_{m=1}^{N} W_m \left(\frac{\partial I_m}{\partial C_{kl}} + \frac{\partial I_m}{\partial C_{lk}} \right) \tag{8}$$

where W_m denotes $\partial W/\partial I_m$ and

$$\begin{aligned} &\frac{\partial I^a_4}{\partial C_{kl}} = a^0_k a^0_l, \ \frac{\partial I^b_4}{\partial C_{kl}} = b^0_k b^0_l, \\ &\frac{\partial I^{ab}_7}{\partial C_{kl}} = -\frac{1}{2}\left[\frac{1}{I^a_4}(I^{ab}_7 + \mathbf{a}_0 \cdot \mathbf{b}_0) a^0_k a^0_l + \frac{1}{I^b_4}(I^{ab}_7 + \mathbf{a}_0 \cdot \mathbf{b}_0) b^0_k b^0_l \right] \\ &+ (I^a_4 I^b_4)^{-\frac{1}{2}} a^0_k b^0_l \end{aligned} \tag{9}$$

Consequently,

$$\begin{aligned} \sigma = I_3^{-\frac{1}{2}} \Bigg[&\left(2\frac{\partial W^F_a}{\partial I^a_4} - \frac{1}{I^a_4}(I^{ab}_7 + \mathbf{a}_0 \cdot \mathbf{b}_0) \frac{\partial W^{FF}_{ab}}{\partial I^{ab}_7} \right) \mathbf{a} \otimes \mathbf{a} \\ &+ \left(2\frac{\partial W^F_b}{\partial I^b_4} - \frac{1}{I^b_4}(I^{ab}_7 + \mathbf{a}_0 \cdot \mathbf{b}_0) \frac{\partial W^{FF}_{ab}}{\partial I^{ab}_7} \right) \mathbf{b} \otimes \mathbf{b} \\ &+ \frac{\partial W^{FF}_{ab}}{\partial I^{ab}_7} \frac{1}{\sqrt{I^a_4 I^b_4}} (\mathbf{a} \otimes \mathbf{b} + \mathbf{b} \otimes \mathbf{a}) \Bigg] \end{aligned} \tag{10}$$

In Eq. (7), the tensile behavior of a woven fabric is assumed as a uni-axial phenomenon in the current study. Several works have shown that the weaving of the warp and weft yarns leads to a biaxial tensile behavior [25,26]. To take into account the biaxial tensile behavior, the fiber stretch energy function W^F should be modified as a function of both warp and weft yarn stretch ratios. As a simplification, in the

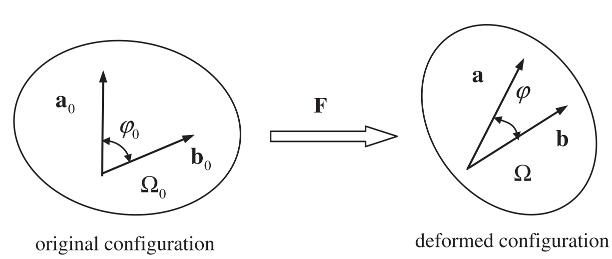

Fig. 1. Geometric description of fiber–fiber interaction.