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Postbuckling of nanotube-reinforced composite cylindrical shells under combined axial and radial mechanical loads in thermal environment

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ABSTRACT

A postbuckling analysis is presented for nanocomposite cylindrical shells reinforced by single-walled carbon nanotubes (SWCNTs) subjected to combined axial and radial mechanical loads in thermal environment. Two types of carbon nanotube-reinforced composite (CNTRC) shells, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. The governing equations are based on a higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity. The thermal effects are also included and the material properties of CNTRCs are assumed to be temperature-dependent. A boundary layer theory and associated singular perturbation technique are employed to determine the buckling loads and postbuckling equilibrium paths. The numerical illustrations concern the postbuckling behavior of perfect and imperfect, FG-CNTRC cylindrical shells under combined action of external pressure and axial compression for different values of load-proportional parameters. The results for UD-CNTRC shell, which is a special case in the present study, are compared with those of the FG-CNTRC shell.

1. Introduction

Carbon nanotubes (CNTs) were discovered by Iijima in 1991 [1] when he was examining carbon materials under an electron microscope. CNTs have fascinated research scientists and engineers due to their remarkable mechanical, electrical and thermal properties and have lead to numerous studies on their properties and applications in the past two decades. CNTs have extremely high tensile strength and stiffness [2,3] that make them to be an ideal material for the reinforcement of polymer composites. The CNT fibers may be aligned in one direction in composites to maximize the effectiveness of CNTs and this has been fabricated in two recent research studies [4,5]. Carbon nanotube-reinforced composites (CNTRCs) possess advanced mechanical properties such as high strength, high stiffness and light weight which can be applied as layers in advanced laminated structures. Unlike the carbon fiber-reinforced composites, the CNTRCs can only contain a low percentage of CTNs (2–5% by weight) [6–9] as more volume fraction in CNTRCs can actually cause the deterioration of their mechanical properties [10]. To overcome this limitation, Shen [11] firstly introduced the functionally graded concept to CNTRCs to effectively make use of CNTs in the composite and studied the nonlinear bending behavior of CNTRC plates. He observed that the load-bending moment curves of the plates can be considerably improved through the use of a functionally graded distribution of aligned CNTs in the matrix. A functionally graded CNT reinforced aluminum matrix composite was recently fabricated by a powder metallurgy route to support the concept of functionally graded materials in the nanocomposites [12]. Applying the concept of functionally graded materials to the nanocomposites, Shen and his co-authors [13–22] studied the postbuckling and nonlinear vibration of CNTRC plates and shells with the linear functionally graded CNT reinforcements. They found that while the nonlinear vibration frequencies, buckling load as well as postbuckling strength of the plate/shell structures under mechanical load are increased substantially due the CNT reinforcement, the thermal buckling of the same plate/shell structure has only gain marginally improvement. Other researchers have also examined the mechanical responses of functionally graded CNTRC beams [23–27], plates [28–33], shells [34] and panels [35,36] under various loading conditions. It is worth noting that, like functionally graded ceramic–metal beams and plates with simply supported boundary conditions, the bifurcation buckling does not exist due to the stretching-bending coupling effect for simply supported unsymmetric functionally graded CNTRC beams and plates subjected to in-plane compression. In the above mentioned studies

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[23–36], most investigations are made on linear static and/or dynamic problems. However, relatively few have been made on nonlinear problems of functionally graded CNTRC structures. Among those, Yang and his co-authors [23,24] studied the nonlinear vibration behavior of functionally graded CNTRC beams with or without piezoelectric layers. Lei et al. [30] performed the large deflection analysis of functionally graded CNTRC plates by using the element-free kp-Ritz method. Kaci et al. [32] studied the nonlinear cylindrical bending of functionally graded CNTRC plates subjected to uniform pressure in thermal environments.

The postbuckling behaviors of CNTRC cylindrical shells subjected to either axial compression or lateral pressure in thermal environments were studied by Shen [19,20]. In the present work, we focus our attention on the postbuckling behavior of CNTRC cylindrical shells under combined action of external pressure and axial compression. The material properties of CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model in which the CNT efficiency parameter is estimated by matching the elastic modulus of CNTRCs observed from the molecular dynamics (MD) simulation results with the numerical results obtained from the extended rule of mixture. The governing equations are based on a higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity and include thermal effects. A boundary layer theory and associated singular perturbation technique are employed to determine the buckling loads and postbuckling equilibrium paths. The nonlinear prebuckling deformations and initial geometric imperfections of the shell are both taken into account. The numerical illustrations show the full nonlinear postbuckling response of FG-CNTRC cylindrical shells subjected to combined axial and radial mechanical loads and under different sets of environmental conditions.

2. Multi-scale model for functionally graded CNTRC shells under combined loading

Consider a CNTRC cylindrical shell with mean radius R, length L and thickness h. The shell is referred to a coordinate system (X,Y,Z) in which X and Y are in the axial and circumferential directions of the shell and Z is in the direction of the inward normal to the middle surface. The corresponding displacements are designated by $\overline{U}$, $\overline{V}$ and $\overline{W}$. $\overline{\Psi}_x$ and $\overline{\Psi}_y$ are the rotations of the normals to the middle surface with respect to the Y and X axes, respectively. The origin of the coordinate system is located at the end of the shell on the middle plane. The shell is assumed to be geometrically imperfect, exposed to elevated temperature, and subjected to combined uniform external pressure q and axial load P_0. Denote the initial geometric imperfection by $\overline{W}^*(X,Y)$ and let $\overline{F}(X,Y)$ be the stress function for the stress resultants defined by $\overline{N}_x = \overline{F}_{,YY}$, $\overline{N}_y = \overline{F}_{,XX}$ and $\overline{N}_{xy} = -\overline{F}_{,XY}$, where a comma denotes partial differentiation with respect to the corresponding coordinates.

Based on Sanders shell theory, Reddy and Liu [37] developed a simple higher order shear deformation shell theory. This theory assumes that the transverse shear strains are parabolically distributed across the shell thickness. The advantages of this theory over the first order shear deformation theory are that the number of independent unknowns ($\overline{U}$, $\overline{V}$, $\overline{W}$, $\overline{\Psi}_x$ and $\overline{\Psi}_y$) is the same as in the first order shear deformation theory, and no shear correction factors are required. Based on Reddy's higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity and including thermal effects, the governing differential equations for a FG-CNTRC cylindrical shell can be derived in terms of a stress function $\overline{F}$, two rotations $\overline{\Psi}_x$ and $\overline{\Psi}_y$, and a transverse displacement $\overline{W}$, along with the initial geometric imperfection $\overline{W}^*$. They are

$$\widetilde{L}_{11}(\overline{W}) - \widetilde{L}_{12}(\overline{\Psi}_x) - \widetilde{L}_{13}(\overline{\Psi}_y) + \widetilde{L}_{14}(\overline{F}) - \widetilde{L}_{15}(\overline{N}^T) - \widetilde{L}_{16}(\overline{M}^T) - \frac{1}{R}\overline{F}_{,XX} = \widetilde{L}(\overline{W} + \overline{W}^*, \overline{F}) + q \tag{1}$$

$$\widetilde{L}_{21}(\overline{F}) + \widetilde{L}_{22}(\overline{\Psi}_x) + \widetilde{L}_{23}(\overline{\Psi}_y) - \widetilde{L}_{24}(\overline{W}) - \widetilde{L}_{25}(\overline{N}^T) + \frac{1}{R}\overline{W}_{,XX} = -\frac{1}{2}\widetilde{L}(\overline{W} + 2\overline{W}^*, \overline{W}) \tag{2}$$

$$\widetilde{L}_{31}(\overline{W}) + \widetilde{L}_{32}(\overline{\Psi}_x) - \widetilde{L}_{33}(\overline{\Psi}_y) + \widetilde{L}_{34}(\overline{F}) - \widetilde{L}_{35}(\overline{N}^T) - \widetilde{L}_{36}(\overline{S}^T) = 0 \tag{3}$$

$$\widetilde{L}_{41}(\overline{W}) - \widetilde{L}_{42}(\overline{\Psi}_x) + \widetilde{L}_{43}(\overline{\Psi}_y) + \widetilde{L}_{44}(\overline{F}) - \widetilde{L}_{45}(\overline{N}^T) - \widetilde{L}_{46}(\overline{S}^T) = 0 \tag{4}$$

Note that the geometric nonlinearity in the von Kármán sense is given in terms of $\widetilde{L}()$ in Eqs. (1) and (2), and the other linear operators $\widetilde{L}_{ij}()$ are defined as in [19]. It is worthy to note that the governing differential Eqs. (1)–(4) for a FG-CNTRC cylindrical shell are identical in form to those of unsymmetric cross-ply laminated shells.

In the above equations, $\overline{N}^T$, $\overline{M}^T$, $\overline{S}^T$, and $\overline{P}^T$ are the forces, moments and higher order moments caused by elevated temperature, and are defined by

$$\begin{bmatrix} \overline{N}_x^T & \overline{M}_x^T & \overline{P}_x^T \\ \overline{N}_y^T & \overline{M}_y^T & \overline{P}_y^T \\ \overline{N}_{xy}^T & \overline{M}_{xy}^T & \overline{P}_{xy}^T \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} A_x \\ A_y \\ A_{xy} \end{bmatrix} (1, Z, Z^3)\Delta T\, dZ \tag{5a}$$

$$\begin{bmatrix} \overline{S}_x^T \\ \overline{S}_y^T \\ \overline{S}_{xy}^T \end{bmatrix} = \begin{bmatrix} \overline{M}_x^T \\ \overline{M}_y^T \\ \overline{M}_{xy}^T \end{bmatrix} - \frac{4}{3h^2} \begin{bmatrix} \overline{P}_x^T \\ \overline{P}_y^T \\ \overline{P}_{xy}^T \end{bmatrix} \tag{5b}$$

where $\Delta T = T - T_0$ is the temperature rise from some reference temperature T_0 at which there are no thermal strains, and

$$\begin{bmatrix} A_x \\ A_y \\ A_{xy} \end{bmatrix} = -\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{11} \\ \alpha_{22} \end{bmatrix} \tag{6}$$

where α_{11} and α_{22} are the thermal expansion coefficients measured in the longitudinal and transverse directions, and $\overline{Q}_{ij}$ are the transformed elastic constants with details being given in [37]. Note that for a FG-CNTRC layer, $\overline{Q}_{ij} = Q_{ij}$ in which

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{16} = Q_{26} = 0, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13}, \quad Q_{66} = G_{12} \tag{7}$$

where E_{11}, E_{22}, G_{12}, ν_{12} and ν_{21} are the effective Young's and shear moduli and Poisson's ratio of the FG-CNTRC layer, respectively.

In order to extend Reddy's theory into the FG-CNTRC shells, it is imperative to estimate the equivalent material properties to account for the impact of CNTs, which is usually done by multi-scale modeling of CNTs and matrix. Several micromechanical models have been developed to predict the effective material properties of CNTRCs, for instance, the Mori–Tanaka model [38,39] and the Voigt model as the rule of the mixture [40,41]. The Mori–Tanaka model is applicable to micro-particles and the rule of mixture is simple and convenient to predict the global material properties and responses of the CNTRC structures. At nanoscale both Mori–Tanaka and Voigt models need to be extended in order to include the small scale effect. It has been shown that the Voigt and Mori–Tanaka models have the same accuracy in predicting the buckling and vibration characteristics of functionally graded