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Trace-based stiffness for a universal design of carbon-fiber reinforced composite structures

Stephen W. Tsai [a], Sangwook Sihn [b], Jose Daniel D. Melo [c,*]

[a] Department of Aeronautics & Astronautics, Stanford University, Durand Building, 496 Lomita Mall, Stanford, CA 94305-4035, USA
[b] University of Dayton Research Institute, 300 College Park, Dayton, OH 45469-0050, USA
[c] Department of Materials Engineering, Federal University of Rio Grande do Norte, Campus Central, 3000, Natal, RN 59078-970, Brazil

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ABSTRACT

A novel invariant-based approach to describe stiffness and strength of carbon-fiber reinforced plastic (CFRP) composites has recently been proposed in the literature. The approach is based on the trace of the plane stress stiffness matrix as a material property. The proposed method allows predicting strength and stiffness of the CFRP composite laminates within 1.5% error using [0] ply test only. The current study evaluates the use of the trace-based approach to set up a universal stress–strain relation among various materials and orthotropic laminates for composite structural components. One such stress–strain relation was evaluated for many CFRP composites using a beam subjected to in-plane and flexural loads. The current approach using the trace was found to be simple and accurate in the optimal design of composite structures once a geometric profile is defined by an isotropic material such as aluminum. Weight savings of composite laminates and structural components with various material and orthotropy combinations over aluminum can simply be determined with the current approach.

1. Introduction

Carbon-fiber reinforced plastic (CFRP) composites have increasingly been used in aircraft and automotive structures because of their superior properties, which includes high strength- and modulus-to-weight ratio, high fatigue resistance and corrosion resistance. The consequential advantages for the aircraft and automotive structures include lower weight, increased fuel efficiency, lower emissions, reduced inspection and maintenance, and increased passenger comfort for fuselage application, increased gas mileage requirement for the automotive application.

The inherent anisotropy of these composite materials – fundamental to design flexibility and superior properties – makes their mechanical characterization complex and time consuming. As a result, an experimental program to characterize the mechanical properties and generate design allowable of the composite materials for aircraft and automotive structures may include thousands of test specimens and cost millions of dollars for years to complete [1]. Likewise, an optimal design of the composite laminates is significantly complicated, if not impossible, due to the large number of combinations of material properties and stacking sequences.

Several studies have been presented in the literature for optimal design of composite structural components [2–7]. A typical optimization approach is to assume a fixed geometry (topology) of the component and focus on optimizing laminate properties. An example of these studies involves the optimization of cross sectional stiffness and inertia properties of a helicopter rotor blade [2]. Other studies addressed the optimal design of composite beams subject to stiffness and aeroelastic constraints [3,4]. Design optimization aiming at weight reduction of structural components is of interest to both industry and academic researchers [5]. One of the previous work conducted the optimization study for maximum stiffness and minimum weight of several composite beams with various laminate thickness, stacking sequence, fiber orientation, and cross sectional shapes [6].

It has been found empirically and demonstrated analytically that any ply groups in a laminate with a certain fiber orientation should be dispersed as much as possible to improve laminate strength and toughness [1]. The more identical sub-laminates repeat, the more the laminates become homogenized. It has been shown that laminates with two plies in the sub-laminate can reach a practical level of homogenization with 32 plies [1]. With

* Corresponding author.
E-mail addresses: stsai@stanford.edu (S.W. Tsai), sangwook@alumni.stanford.edu (S. Sihn), jddmelo@gmail.com (J.D.D. Melo).

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homogenization, the in-plane and flexural stiffnesses of laminates converge. Designing of homogenized composite laminates then becomes analogous to that of isotropic materials such as aluminum. Thus, optimization is greatly simplified with the homogenized orthotropic laminates. For heterogeneous laminates, the enormous number of possible stacking combinations makes the optimum design study extremely difficult, if not impossible.

Recently, a novel invariant-based approach was proposed to predict elastic properties and failure of composite laminates [1,8]. In this approach, the invariant is the trace of the plane-stress stiffness matrix, which was proposed as a material property. The CFRP composite materials were found to share common stiffness properties if these properties are normalized by their respective trace of the plane-stress stiffness matrix. A "master ply" was defined by taking average values of these normalized properties for a wide variety of the CFRP composites with a small coefficient of variation, in fact, less than one half of an usual experimental accuracy of 3% for laminate stiffness data.

The purpose of the present work is to extend the use of the trace-based approach as a scale factor to the optimal design of structural components made of composite materials. The concept of homogenized laminates is used to directly compare the optimal solutions between isotropic and composite laminates. With the trace-based scaling factor, the weight savings of composite structures as compared to isotropic materials can easily be demonstrated for various materials, stacking sequences and profile topologies.

2. Trace-based scaling

The CFRP composite materials have been found to share common stiffness properties if these properties are normalized by their respective trace of the stiffness matrix, Tr [Q], [8]. The normalized stiffness factors for fifteen CFRP are listed in Table 1. These normalized properties are very similar, particularly in the longitudinal stiffness, parallel to the fiber orientation. The average values of these factors have been used to define a "master ply". The trace value for the master ply is unity, as shown in Table 1.

Effective elastic properties of multidirectional laminates and their normalized properties can be calculated using the classical laminated plate theory. Table 2 lists the effective longitudinal Young's modulus normalized by the trace, $E_1{}^*$, of various stacking sequences of laminates for the fifteen materials in Table 1. The $E_1{}^*$ is denoted as a laminate factor.

While the laminate factors of typical laminates such as $[0_2/\pm45]$, $[0_5/\pm45_2/90]$ and $[0/\pm45/90]$ are less than 0.52, the second generation of laminates with thin plies and shallow angles such as $[\pm12.5]$ and $[0_2/\pm25]$ results in a significant increase in $E_1{}^*$. In this case, ply material stays the same; only the number of ply angles reduces from 4 to 3 or 2, and the off-axis angles are shallower.

If the laminate is subjected to the in-plane axial loading, the ratio between the structural weight of the CFRP material and that of the isotropic material, such as aluminum, is given by

$$\frac{W_c}{W_i} = \frac{\rho_c A_c}{\rho_i A_i}, \tag{1}$$

where W, ρ and A are weight, volumetric mass density and cross-sectional area, respectively. Subscripts c and i stand for the CFRP composite and the isotropic materials, respectively.

Considering two structural components of the same axial rigidity given by

$$EA = Tr_c E^*_{1c} A_c = Tr_i E^*_{1i} A_i, \tag{2}$$

where Tr is the trace of the in-plane stiffness matrix and E^*_1 is the normalized longitudinal Young's modulus (laminate factor) for the corresponding laminates, Eq. (1) becomes

$$\frac{W_c}{W_i} = \frac{\rho_c Tr_i E^*_{1i}}{\rho_i Tr_c E^*_{1c}}. \tag{3}$$

In Eq. (3), density and trace represent the material properties, while the laminate factor represents a geometric parameter related to the stacking sequence. For comparing the composite laminates with the metals, Eq. (3) can further be simplified if the following properties are used: 1) aluminum, titanium and steel have the same value of trace normalized by density (79 GPa cm^3/g); 2) the normalized Young's modulus of all isotropic materials is 0.337 for a Poisson's ratio of 0.3. In this case, Eq. (3) becomes

$$\frac{W_c}{W_i} = \frac{26.6\rho_c}{Tr_c E^*_{1c}}. \tag{4}$$

The units of Tr and ρ are GPa and g/cm^3, respectively.

For the flexural loading, considering a beam having a rectangular cross-section with a given width (b) and height (h), and assuming the same beam width for the two materials, i.e., $b_c = b_i$,

Table 1
Engineering constants and plane-stress stiffness components normalized by trace for various CFRP composites.

Material	E_x (GPa)	E_y (GPa)	ν_x	E_s (GPa)	$Q_{xx}{}^*$	$Q_{yy}{}^*$	$Q_{xy}{}^*$	$Q_{ss}{}^*$	Tr (GPa)
IM6/epoxy	203	11.20	0.32	8.40	0.8791	0.0485	0.0155	0.0362	232
IM7/977-3	191	9.94	0.35	7.79	0.8825	0.0459	0.0161	0.0358	218
T300/5208	181	10.30	0.28	7.17	0.8805	0.0501	0.0140	0.0347	206
IM7/MTM45	175	8.20	0.33	5.50	0.9014	0.0422	0.0139	0.0282	195
T800/Cytec	162	9.00	0.40	5.00	0.8955	0.0497	0.0199	0.0274	183
IM7/8552	159	8.96	0.32	5.50	0.8888	0.0501	0.0160	0.0306	180
T800S/3900	151	8.20	0.33	4.00	0.9034	0.0491	0.0162	0.0238	168
T300/F934	148	9.65	0.30	4.55	0.8878	0.0579	0.0174	0.0271	168
T700 C-Ply 64	141	9.30	0.30	5.80	0.8713	0.0575	0.0172	0.0356	163
AS4/H3501	138	8.96	0.30	7.10	0.8567	0.0556	0.0167	0.0438	162
T650/epoxy	139	9.40	0.32	5.50	0.8724	0.0590	0.0189	0.0343	160
T4708/MR60H	142	7.72	0.34	3.80	0.9029	0.0491	0.0167	0.0240	158
T700/2510	126	8.40	0.31	4.20	0.8827	0.0588	0.0182	0.0292	144
AS4/MTM45	127	7.93	0.30	3.60	0.8938	0.0558	0.0167	0.0252	143
T700 C-Ply 55	121	8.00	0.30	4.70	0.8746	0.0578	0.0173	0.0338	139
Std dev	24.6	1.0	0.029	1.5	0.0132	0.0053	0.0016	0.0056	
Coeff var %	16.0	10.9	9.0	27.2	1.5	10.1	9.6	17.9	
Master ply					0.8849	0.0525	0.0167	0.0313	1

Note: $Q_{ij}{}^*$ are the plane stress stiffness components normalized by the trace.