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An improved analytical model for shear modulus of fiber reinforced laminates with damage

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ABSTRACT

When loaded normal to the fiber direction, transverse cracks often form in the matrix of polymeric composites. Transverse softening has been widely studied, where agreement between models and experiment is common. By comparison, shear softening from transverse cracks has received little attention, and tends not to agree with predictions. The discrepancy between models and observations may be due to the presence of a traction between the crack surfaces. A closed form solution is proposed for the shear stress–strain field of a cracked laminate by replacing the cracks with cohesive zones. The constitutive equations of the crack laminate were derived including the effects of internal tractions and transverse stress on the shear modulus. The analytical solution resulted in good agreement with experimental measurements.

1. Introduction

Matrix cracking is usually the first dominant damage mode occurring in fiber reinforced composite materials. The first effect of matrix cracks is degradation in the thermoelastic properties of the cracked plies, including changes in stiffness, Poisson ratio, and thermal expansion coefficients. Failure in the fiber direction is usually catastrophic, while degradation factors are applied when failure occurs in the matrix. Cracks in the matrix parallel to the fibers change the transverse and shear stiffness. The transverse stiffness reduction can be predicted using analytical or semi-analytical solutions. Analogous models for the shear modulus reduction have been developed; however, these methods have not been experimentally verified [1,2].

The stiffness reduction of damaged laminates has been investigated by many researchers. The approaches may be categorized as analytical, semi-analytical and numerical models. Among these approaches, analytical models are powerful tools for design of the composite structures. However, the analytical models embody simplifications and assumptions which need to be verified by experiments. Hashin [3,4] analyzed the stiffness reduction of cracked cross ply laminates by a variational method on the basis of the principle of minimum complementary energy. Axial stiffness reductions were in good agreement with experimental data but shear modulus reduction was not evaluated.

Tan and Nuismer [5] and Nuismer and Tan [6] proposed an analytical solution based on a shear lag model for progressive matrix cracking in a composite laminate. The closed form solutions were obtained for laminate stiffness and Poisson's ratio as a function of crack density. The laminate damaged axial stiffness, $\overline{C}_{11}$ and shear modulus, $\overline{C}_{66}$ were defined for a cross-ply laminate as

$$\overline{C}_{11} = \frac{h^a C_{11}^a + h^b C_{11}^b}{\beta_2 (h^a + h^b)} \tag{1}$$

$$\overline{C}_{66} = \frac{h^a C_{66}^a + h^b C_{66}^b}{\beta_4 (h^a + h^b)} \tag{2}$$

where h, C_{11}, C_{66} are the thickness, average axial stiffness and average shear stiffness for the layer, respectively. The superscripts a and b refer to the 90° and supporting layers, respectively. The definition of the parameters β_2 and β_4 can be found in [5]. The model was compared with experimental data for transverse stiffness and Poisson's ratio. Zhang et al. [7] proposed a semi-analytic model for predicting the thermoplastic property degradation in general symmetric laminates with uniform ply cracks in some or all of the 90° layers. The application of the methodology was shown by numerical examples of material stiffness degradation. The transverse stiffness and Poisson's ratio were compared with experimental data.

Knops and Bogle [8] used a tubular specimen to compare shear modulus degradation with Puck's failure theory. In contrast to Puck's assumption that matrix cracks have the same effect on the

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transverse and shear modulus, their data showed that the shear modulus was less affected than the transverse modulus. While Knops didn't compare his results with analytical models, he postulated that friction on the crack surface as the source of discrepancy. Smith and Salavatian [9–11] proposed an experimental method to measure material stiffness degradation due to matrix damage. A modified Iosipescu coupon was designed to study the evolution of shear and transverse damage including their mutual interaction. The layup and coupon geometry were selected to control the severity of the damage and allow the measurement of shear and transverse stiffness degradation experimentally. The proposed method showed good agreement with results from tubular specimens and has advantages of simplified specimen fabrication using standard test fixtures. The results provided the first experimental comparison of shear modulus reduction, from transverse damage, to predictive models. The results were compared with existing analytical and numerical models, which over estimated the observed shear modulus reduction.

In the following, an elasticity solution is proposed for the shear stress–strain field of a transversely cracked laminate. The formulation of the analytical model considers the effects of internal tractions over the crack faces by assuming a cohesive zone instead of a crack and describes the constitutive equation for the whole laminate.

2. Model formulation

Consider a cracked fiber reinforced laminate in the global coordinate (x–y–z) system under general in-plane loading as shown in Fig. 1. The laminate is made of an orthotropic homogeneous, linear elastic material. In the present model, the laminate consists of a central cracked 90° lamina surrounded on both sides by either a $[0]_n$ supporting laminate or a balanced and symmetric angle-ply $[\pm\theta]_{ns}$ supporting laminate The laminate has constant thickness h, which is composed of constant thickness h^a and h^b for the 90° lamina and supporting layer, respectively. Assuming that the distance between cracks is a uniform length, $2l$, a one-quarter unit cell (shown in Fig. 2) was used to solve the displacement, strain and stress field.

2.1. Basic assumptions

The basic constitutive equation for thermo-elastic analysis of a composite material is

$$\{\sigma\} = [C](\{\varepsilon\} - \{varepsilon_T\}) \tag{3}$$

where $[C]$, $\{\varepsilon\}$, $\{\varepsilon_T\}$, and $\{\sigma\}$ are the stiffness, mechanical strain, thermal strain, and stress tensors, respectively. Eq. (3) may be written for each lamina in matrix form as,

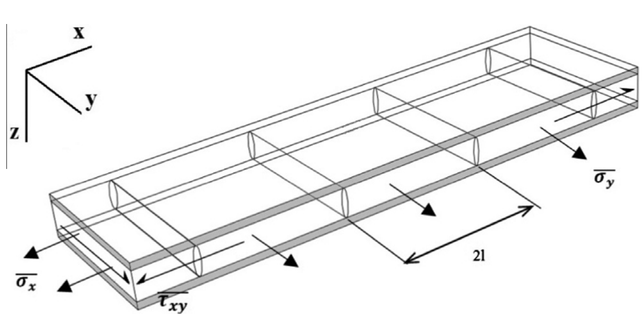

Fig. 1. Cross-ply laminate with cracked 90° lamina.

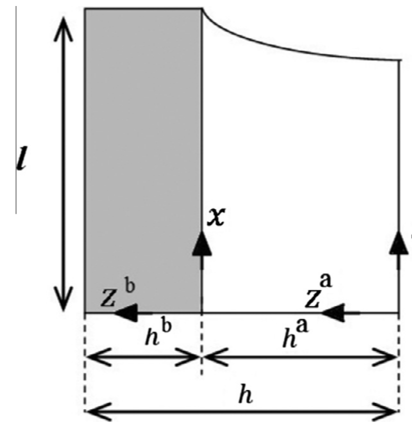

Fig. 2. 2D unit cell and local coordinate systems for each ply.

$$\begin{pmatrix}\sigma_x^i\\ \sigma_y^i\\ \sigma_z^i\\ \tau_{yz}^i\\ \tau_{xz}^i\\ \tau_{xy}^i\end{pmatrix} = \begin{bmatrix} C_{11}^i & C_{12}^i & C_{13}^i & 0 & 0 & 0\\ C_{12}^i & C_{22}^i & C_{23}^i & 0 & 0 & 0\\ C_{13}^i & C_{23}^i & C_{33}^i & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44}^i & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55}^i & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}^i \end{bmatrix} \begin{pmatrix}\epsilon_x^i - \epsilon_{xT}^i\\ \epsilon_y^i - \epsilon_{yT}^i\\ \epsilon_z^i - \epsilon_{zT}^i\\ \gamma_{yz}^i\\ \gamma_{xz}^i\\ \gamma_{xy}^i\end{pmatrix} \tag{4}$$

where the superscript i denotes central and the supporting layer for a and b, respectively. The current solution is based on orthotropic supporting plies, such as a balanced symmetric layup, $[\pm\theta]_{ns}$ which is widely used in practical applications. In general the supporting lamina can be anisotropic which adds normal-shear coupling terms in Eq. (4) and subsequent equations. This work considers a cross ply laminate which simplifies the formulation for this demonstrative application.

The equilibrium equations may be expressed as

$$\begin{cases} \dfrac{\partial\sigma_x^i}{\partial x} + \dfrac{\partial\tau_{xy}^i}{\partial y} + \dfrac{\partial\tau_{xz}^i}{\partial z} = 0\\ \dfrac{\partial\tau_{xy}^i}{\partial x} + \dfrac{\partial\sigma_y^i}{\partial y} + \dfrac{\partial\tau_{yz}^i}{\partial z} = 0\\ \dfrac{\partial\tau_{xz}^i}{\partial x} + \dfrac{\partial\tau_{yz}^i}{\partial y} + \dfrac{\partial\sigma_z^i}{\partial z} = 0 \end{cases} \tag{5}$$

Assuming a uniform load and geometric symmetric in the y direction, displacements can be expressed as

$$\begin{cases} u^i = u^i(x,z)\\ v^i = v^i(x,z) + \bar{\epsilon}_0 y\\ w^i = w^i(x,z) \end{cases} \tag{6}$$

The strains may be defined in terms of displacements as

$$\begin{aligned} &\epsilon_x^i = \frac{\partial u^i}{\partial x} && \gamma_{yz}^i = \frac{\partial w^i}{\partial y} + \frac{\partial v^i}{\partial z} = \frac{\partial v^i}{\partial z}\\ &\epsilon_y^i = \frac{\partial v^i}{\partial y} = \bar{\epsilon}_0 && \gamma_{xz}^i = \frac{\partial u^i}{\partial z} + \frac{\partial w^i}{\partial x}\\ &\epsilon_z^i = \frac{\partial w^i}{\partial z} && \gamma_{xy}^i = \frac{\partial v^i}{\partial x} + \frac{\partial u^i}{\partial y} = \frac{\partial v^i}{\partial x} \end{aligned} \tag{7}$$

The constitutive and equilibrium equations were averaged in the z direction according to

$$(^{-})^i = \frac{1}{h^i}\int_{h^i} ()^i dz \tag{8}$$

where () represents the averaged variable. A linear distribution of the out of plane shear stress was adopted in this solution as