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Modelling of textile composites with fibre strength variability

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ABSTRACT

Scatter in composite mechanical properties is related to variabilities occurring at different scales. This work attempts to analyse fibre strength variability numerically from micro to macro-scale taking into account the size effect and its transition between scales. Two micro-mechanical models based on the Weibull distribution were used within meso-scale finite element models of fibre bundles which were validated against experimental results. These models were then implemented in a meso-scale model of an AS4 carbon fibre plain weave/vinyl ester textile composite. Monte Carlo simulations showed that fibre strength variability has a limited effect on the strength of the textile composite at the meso-scale and introduces variability of less than 2% from the mean value. Macro-scale strength based on the predicted meso-scale distribution was lower than the strength of the composite without variability by 1–4% depending on the model. The presented multi-scale approach demonstrates that a wide fibre strength distribution leads to a narrow distribution of composite strength and a shift to lower mean values.

1. Introduction

Composite mechanical properties are highly scattered due to the presence of variabilities [1], e.g. the tensile strength of unidirectional (UD) composites can have coefficients of variation (CoV) of up to 5% [2]. Defects induced by manufacturing (e.g. yarn waviness or variable ply placement) or variations in constituent properties affect composite properties. According to the multi-scale approach, uncertainties are divided into groups by length scale [2]. Micro-scale variabilities include packing of fibres within yarns, fibre waviness [3], voids between fibres and variability of constituent properties; meso-scale variabilities include variation of yarn path [4], size and shape of yarn cross-section, nesting and voids between yarns. All of these cause variations in local moduli (and therefore global stiffness), local strength (hence global strength) and local component shape distortions (hence global geometry), and it is not known *a priori* which of these are significant. Variability of fibre strength is well-known to affect composite properties and is well-studied [5]. However, no published studies explicitly link the distribution of fibre strength to the strength distribution of a woven composite.

Many analytical and numerical methods are based on the multi-scale approach, whereby a complex structure is divided into hierarchical sub-structures according to characteristic length. A heterogeneous medium at one scale is replaced by a homogeneous medium with the same properties at a higher scale. Homogenisation is usually based on the assumption of ideal periodicity at all levels and subsequent representation of a composite as a periodic unit cell. This approach has shown good results [6–8] despite controversy regarding the variability for real structures.

At the micro-scale the strength of single fibres can have a CoV of up to 20% and has a strong length dependence (strength can drop by 10% when length is increased by a factor of 10) [9]. This dependency and distribution are usually described by a two-parameter Weibull distribution with a length scale effect [5]. However, additional parameters are often required for correct description of the length effect [9].

The next step in multi-scale modelling is prediction of the strength of an impregnated fibre bundle or UD composite at the meso-scale. Several approaches can be considered. The Equal Load Sharing (ELS) concept postulates that the load from a broken fibre is equally distributed over all surviving fibres. This was used by Daniels [10] to derive mean strength and its distribution for an unimpregnated fibre bundle. A development of the model known as chain-of-bundles was employed for prediction of the strength of long fibre composites [11]. A drawback of this concept for an impregnated bundle is that it does not account for the unequal redistribution of stresses between fibres. GLS (Global Load Sharing) models assume an unloading zone at each side of fibre breakage. Theoretical predictions with various modifications are possible for a regular fibre arrangement. It was shown that both ELS and GLS models give close results once correct normalising constants are chosen [5]. Strength predicted by both approaches can be

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approximated by a normal distribution [5]. Unlike the ELS, the Local Load Sharing (LLS) concept assumes that the load from a broken fibre is distributed unequally to a number of neighbouring fibres according to a sharing rule [12,13]. The number of neighbouring fibres that take the load depends on the properties of fibres and matrix and on the chosen theory. A number of analytical LLS models [5,14] are able to predict final strength and its distribution. Computational LLS enables direct numerical simulations to be performed. Okabe and Takeda [15] used a spring model in conjunction with a shear lag law to simulate the strength of UD composites. On the other hand, recent research [16] shows the importance of realistic geometry (i.e. fibre packing) in predicting the stress–strain state of a UD composite in the case of fibre breakage. Finite element (FE) analysis was used to obtain the strength of a bundle of randomly packed fibres whose strength followed a Weibull distribution [17]. These models were able to capture the process of damage propagation or a realistic stress–strain state of the fibre array during fibre failure. However, an implementation of these micro-scale models at the meso-scale is not feasible.

The next step is meso-scale modelling of textile composites using properties obtained at the previous stage. Ismar et al. [18] modelled an SiC/SiC woven composite with variability of yarn strength using FE analysis and a Monte Carlo method, varying strength in every element following a Weibull distribution and implementing a size effect. This showed the significant influence (about 10% reduction when a Weibull shape parameter was halved) of variability in impregnated bundle strength on tensile strength of woven composites. However, this study did not report standard deviation of final strength for a given distribution and predictions were not based on the distribution of single fibre strength.

This paper applies a multi-scale modelling approach for a textile composite with variability in fibre properties. Fibre bundle strength models were chosen and validated against experimental data for UD composites based on single fibre strength distributions. The models ensured correct transition between scales taking into account the size effect which is critical for meso-scale FE modelling. Using the fibre bundle strength model, stochastic FE simulations were performed to determine the distribution of composite mechanical properties.

2. Variability models

2.1. Strength model of single fibre

The Weibull distribution is often used for prediction of single fibre strength [5,19]. Taking into account the length effect, a fibre of length L under tensile stress σ has a cumulative failure probability P_f given by

$$P_f = 1 - \exp(-(L/L_0)(\sigma/\sigma_0)^{\rho}) \tag{1}$$

where σ_0 is the Weibull scale parameter, ρ the shape parameter and L_0 the gauge length.

However, it was found that this approach tended to overestimate the strength of some types of fibres of shorter length [9,15,20], and the experimental fibre strength distribution at different length scales is better described by [9]

$$P_f = 1 - \exp\left(-(L/L_0)^{\alpha}(\sigma/\sigma_0)^{\rho}\right) \tag{2}$$

where α is an additional parameter satisfying $0 < \alpha \leqslant 1$.

This empirical relationship was related to fibre-to-fibre variation of the scale parameter [9]. This was explored by Beyerlein and Phoenix [21] for a bundle consisting of four fibres. This approach, termed Weibull of Weibulls, was extended further by applying it to all fibres in the composite [22]. Then the cumulative probability of fibre failure, P_f, under loading stress σ is

$$P_f = 1 - \exp\left(-(L/L_0)(\sigma/\sigma_0^i)^{\rho'}\right) \tag{3}$$

where L is fibre length, L_0 is the reference gauge length, $\rho' = \rho/\alpha$ is a Weibull shape parameter and the Weibull scale parameter σ_0^i has a cumulative distribution P_{σ_0}

$$P_{\sigma_0} = 1 - \exp\left(-(\sigma_0^i/\bar{\sigma}_0)^{m}\right) \tag{4}$$

where m is a Weibull shape parameter and $\bar{\sigma}_0$ is a scale parameter. Curtin showed that Eqs. (3) and (4) give a strength distribution close to that from Eq. (2), but with parameters measured directly from single fibre tests. Eqs. (3) and (4) are used later to describe distribution of fibre strength in fibre bundles.

2.2. Strength model of impregnated fibre bundle

Analysis of failed UD composites shows that fibre damage tends to cluster before final failure [15]. Here it is assumed that in a full-scale FE model these clusters can be modelled as small bundles or domains (finite elements) whose strength is calculated using a theoretical model. Three approaches were employed for comparison: the ELS concept using direct calculations as described below, the ELS concept in its normal distribution approximation [5,10] and the GLS concept in its approximation as a Gaussian process [5,23,24]. However, the entire approach effectively implements an LLS model due to stress redistributions in an FE model which follow element failure.

To find the strength of an individual element a bundle of N fibres is considered. For the ELS approach the stress S^i prior to i-th fibre failure is [10]:

$$S^i = \frac{V_f}{N}(N - i + 1)S_f^i + (1 - Vf)S_m', \tag{5}$$

where V_f is fibre volume fraction, $S_m' = E_m S_f^i / E_f$ is stress in the matrix at the fibre failure strain and S_f^i., $i = 1 \ldots N$ are the strengths of individual fibres, calculated with Eqs. (3) and (4). The second term in Eq. (5) accounts for matrix stress contribution.

Eq. (5) defines a series of stresses corresponding to progressive failure of fibres in the bundle. The ultimate bundle strength is then defined as $\max(S^i)$. It was shown by Daniels [10] that the strength of an infinitely large bundle tends to a normal distribution.

A more realistic GLS concept was also employed as an alternative method for determining the bundle (finite element) strength. It assumes that load is redistributed through shear after a single fibre break. The strength distribution along the length of the bundle predicted by this concept was shown to be asymptotically close to a Gaussian process which mean, standard deviation and covariance function can be found in [24]. The comparison with the ELS model was shown to be possible when the correct characteristic strength is used for the GLS model [5].

It should be noted that, in all the models, fibres within the impregnated bundle are assumed to be perfectly aligned with the finite element edges and therefore the size of the element defines its strength through the length and size of the fibre bundle.

2.3. Damage model

A continuum damage mechanics (CDM) approach was used for the impregnated bundle [25]. This suggests linear behaviour until damage initiation, followed by gradual degradation of elastic properties. Five failure modes were considered: longitudinal tension/compression, transverse tension/compression and transverse shear. Damage initiates when one of the damage variables D_i