Composites Science and Technology 103 (2014) 119–126

Contents lists available at ScienceDirect

Composites Science and Technology

journal homepage: www.elsevier.com/locate/compscitech

Toughening by plastic cavitation around cylindrical particles and fibres

J.G. Williams [a,c], B.R.K. Blackman [a,*], H. Steininger [b], K. Zuo [a]

[a] Imperial College London, Department of Mechanical Engineering, City & Guilds Building, South Kensington Campus, London SW7 2AZ, UK
[b] BASF SE, Materials Physics and Analytics, Germany
[c] AMME Department, University of Sydney, Australia

ARTICLE INFO

Article history:
Received 7 March 2014
Received in revised form 7 July 2014
Accepted 10 July 2014
Available online 27 July 2014

Keywords:
A. Nano composites
B. Fracture toughness
C. Crack

ABSTRACT

A model for toughening from spherical particles is extended to cylindrical rods and fibres. The equations describing the plastic cavitation are of similar form to those of spheres. The results are used to analyse the toughening of an epoxy resin using carbon nano tubes and ZnO nano-rods in a thermoplastic. Both quasi-static fracture and fatigue are included. The effects of fibre size distribution and agglomeration are also discussed. It is proposed that the surface energy at debonding is some fraction of the matrix toughness which is governed by the particle and plastic zone sizes. Rod-like nano particles appear to be more effective as tougheners although agglomeration is the major problem.

1. Introduction

It has become common place for polymers to be modified with micron or nano-scale particles for the enhancement of mechanical properties. The present paper considers the effects of toughening with particles of various sizes and aspect ratios. A model is developed for toughening with cyclindrical rods and fibres and is an extension of an earlier model for toughening with spherical particles [1]. In this introduction we summarise the approach previously taken for the analysis of toughening with spheres, and then in the next section extend this to toughening with cylindrical rods and fibres. This approach allows for a clear comparison of the sphere versus rod analyses. Section 3 considers the effects of a distribution of particle sizes and agglomeration. In Section 4 the analysis is applied to a number of materials where data has appeared in the literature and results are then discussed.

An earlier paper [1] described the toughening mechanism of plastic void growth from adhered spherical particles. The mechanism is of interest since it was shown that the behaviour was different for nano-scale particles (radius 10 nm) when compared with the microscale (radius 10 μm). The analysis modelled a single spherical particle of radius r_o subjected to a hydrostatic stress field and showed that the particles debond when the interfacial stress, σ_c, is given by:

$$G_a = \left(\frac{1+\upsilon}{4}\right)\frac{\sigma_c^2 r_o}{E} \quad (1)$$

where G_a is the interfacial energy, E and υ are Young's modulus and Poisson's ratio of the matrix respectively. For a matrix with a yield stress of σ_Y this dedonding gives a plastic energy, per unit area of the particle surface, of:

$$G_p = \frac{1}{6}\frac{(1+\upsilon)^2}{(1-\upsilon)}\cdot\frac{\sigma_c^2 r_o}{E}\left(\frac{e^{x_s-1}}{x_s}-1\right), \quad x_s = \frac{1}{2}\left(\frac{1+\upsilon}{1-\upsilon}\right)\cdot\frac{\sigma_c}{\sigma_Y} \quad (2)$$

and $G_p = 0$ for $x_s = 1$. The total energy dissipated in debonding a single particle is thus:

$$4\pi r_o^2(G_p+G_a) = \frac{2\pi}{3}\left(\frac{(1+\upsilon)^2}{(1-\upsilon)}\frac{\sigma_c^2 r_o^3}{E}\left[\frac{e^{x_s-1}}{x_s}-\frac{(5\upsilon-1)}{2(1+\upsilon)}\right]\right) \quad (3)$$

The analysis proceeds by assuming a uniform array of particles so that each particle is at the centre of a cube of scale length, l, given by the volume fraction, ϕ such that:

$$\phi = \frac{4\pi r_o^3}{3l^3} \quad (4)$$

The number of such cells, N, above and below the fracture surface is given by the size of the zone, c, in which the stress is greater than σ_c, where c is given by:

$$c = \frac{1}{2\pi}\frac{EG_m}{\sigma_c^2} \quad (5a)$$

and where G_m is the matrix toughness. Thus:

$$N = \frac{2c}{l} = \frac{1}{\pi}\frac{EG_m}{\sigma_c^2 l} \quad (5b)$$

* Corresponding author.
E-mail address: b.blackman@imperial.ac.uk (B.R.K. Blackman).

http://dx.doi.org/10.1016/j.compscitech.2014.07.014
0266-3538/

Nomenclature

A_1, B_1	Lamé constants
c	radius of process zone
E	Young's modulus of the matrix
f_n	proportion of particles with discrete values of r_o across a distribution
G_a	interfacial energy between particle and matrix
G_c	toughness of the composite
G_m	toughness of the matrix
G_p	plastic energy dissipated (per unit area of particle debonded)
G_{th}	threshold value of G in fatigue
k	parameter used in the model to relate G_a to matrix toughness and particle size
l	length of side of representative volume element
l_r	length of a rod-like particle
$l_{r\infty}$	length of a rod-like particle with infinite aspect ratio
N	number of active cells in the particle debonding model
N_∞	value of N when infinite aspect ratio particles are considered
n	parameter used in the model to relate G_a to matrix toughness and particle size
r_m	radius of plane strain plastic zone
r_o	radius of the toughening particle (sphere or rod-like)
$\bar{r}_o$	median value of particle radius in a distribution
r_1	radius of assumed cylinder of matrix surrounding the rod-like particle
S	aspect ratio, ($l_r/2r_o$), =1 for (spheres); =3 (the short rod-like particles considered here); $\rightarrow\infty$ (carbon nanotubes)
u	radial displacement
x_s	critical stress ratio factor for spherical particles
x_r	critical stress ratio factor for rod-like particles
X	toughening factor
X_s	toughening factor for spherical particles
$X_{r\infty}$	toughening factor for rod-like particles (infinite aspect ratio assumed)
X_3	toughening factor for rod-like particles (aspect ratio, $S = 3$)
Y_s	toughening factor for spherical particles when $N = 1$
Y_∞	toughening factor when $N_\infty = 1$ is assumed
Y_{th}	toughening factor for fatigue threshold when $N_\infty = 1$ is assumed
$\varepsilon_{\theta,r,z}$	strain in the hoop, radial and axial directions
ϕ	volume fraction of particles in composite
ϕ_c	lower limit to volume fraction to give $N = 1$
ϕ_∞	volume fraction when infinite aspect ratio rods are considered
ϕ_{rf}	volume fraction for finite, flat-ended rods
$\phi_{c\infty}$	value of ϕ_c for the case of infinite aspect ratio rods
$\bar{\phi}$	volume fraction, below which corresponds to an infinite aspect ratio
σ_c	critical stress on particle–matrix interface for debonding
σ_h	hydrostatic stress in the matrix surrounding the particle
σ_Y	yield stress of the matrix
$\sigma_{\theta,r,z}$	stress in hoop, radial and axial directions
ξ	ratio $(r_o/r_1)^2$
ν	Poisson's ratio of the matrix

and the increase in toughness due to cavitation is given by:

$$\Delta G_c = N\frac{4\pi r_o^2}{l^2}(G_p + G_a) = \frac{1}{2\pi}\frac{(1+\nu)^2}{(1-\nu)}\left(\frac{e^{x_s-1}}{x_s} - \frac{(5\nu-1)}{2(1+\nu)}\right)\phi\cdot G_m \qquad (6)$$

(A loss of area term may also be included as in [1] but it is generally small). Thus the toughness increase is proportional to ϕ and is determined by $x_s > 1$. For $\nu = 1/3$, the stress ratio and toughening factors for spherical particles are given by:

$$x_s = \frac{\sigma_c}{\sigma_Y} \quad \text{and} \quad X_s = \frac{1}{\phi}\left(\frac{G_c}{G_m} - 1\right) = \frac{4}{3\pi}\left(\frac{e^{x_s-1}}{x_s} - \frac{1}{4}\right) \qquad (7)$$

and from Eq. (1) the controlling factor is G_a which is given by:

$$\frac{x_s^2}{3} = \left(\frac{EG_a}{\sigma_Y^2 r_o}\right) \qquad (8)$$

Fig. 1 shows $\frac{EG_a}{\sigma_Y^2 r_o}$ and $x_s = \frac{\sigma_c}{\sigma_Y}$ as functions of X_s. An example is given in [1] for data measured using silica particles with $r_o = 10$ nm at $\Phi = 0.1$ and gives $X_s = 30$ so that $\frac{EG_a}{\sigma_Y^2 r_o} = 18$ and $x_s = 7.3$ which gives $G_a = 0.3$ J/m^2.

The lower limit of this analysis is given when $N = 1$. From Eq. (4) and substitutions from Eqs. (1) and (5b) it can be shown that a lower limit to volume fraction, to give $N = 1$, is given by:

$$\phi = \phi_c = \frac{4\pi}{3}\left(\frac{4\pi}{1+\nu}\frac{G_a}{G_m}\right)^3$$

For $N = 1$ and $\nu = 1/3$,

$$\frac{\Delta G_c}{G_m} = \frac{16\pi}{3}\left(\frac{3}{4\pi}\right)^{2/3}\left(\frac{G_a}{G_m}\right)\left(\frac{e^{x_s-1}}{x_s} - \frac{1}{4}\right) = \phi_c^{1/3}X_s\phi^{2/3}$$

For $\phi < \phi_c$ the toughness has a $\phi^{\frac{2}{3}}$ dependence and a parameter Y may be determined experimentally, i.e.,

$$Y_s = \frac{1}{\phi^{2/3}}\frac{\Delta G_c}{G_m} = \phi_c^{1/3}X_s$$

For the example discussed above, $G_m = 277$ J/m^2 and hence $\phi_c = 4\times10^{-6}$, i.e. very small so the $N > 1$ solution is appropriate. To analyse toughening in fatigue, the matrix value of G, i.e. G_m,

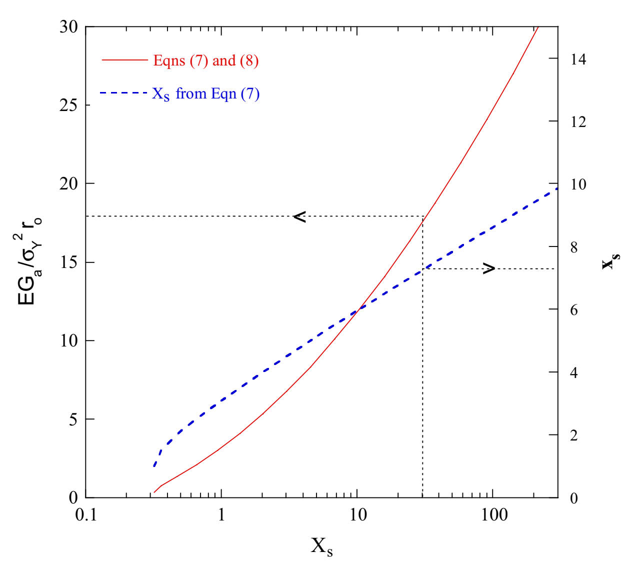

Fig. 1. Toughening factor X_s and critical stress ratio x_s (ordinate) for spherical particles in a matrix of Poisson's ratio $\nu = 1/3$ (the dotted lines are for the case of $X_s \approx 30$ referred to in the text).