Composites Science and Technology 103 (2014) 127–133

Contents lists available at ScienceDirect

Composites Science and Technology

journal homepage: www.elsevier.com/locate/compscitech

Generalized curvature tailoring of bistable CFRP laminates by curing on a cylindrical tool-plate with misalignment

Junghyun Ryu [a], Jong-Gu Lee [a], Seung-Won Kim [b], Je-Sung Koh [b], Kyu-Jin Cho [b], Maenghyo Cho [a,*]

[a] Smart Structures & Design Laboratory, School of Mechanical and Aerospace Engineering/IAMD, Seoul National University, Gwanak 599 Gwanak-ro, Gwanak-gu, Seoul 151-742, Republic of Korea

[b] Biorobotics Laboratory, School of Mechanical and Aerospace Engineering/IAMD, Seoul National University, Gwanak 599 Gwanak-ro, Gwanak-gu, Seoul 151-742, Republic of Korea

ARTICLE INFO

Article history:
Received 28 February 2014
Received in revised form 18 August 2014
Accepted 24 August 2014
Available online 29 August 2014

Keywords:
A. Carbon fibers
B. Non-linear behavior
C. Laminate theory
C. Residual stress

ABSTRACT

Inducing an initial curvature is advantageous for tailoring the curvature of bistable Carbon Fiber Reinforced Prepreg (CFRP) laminate because the final curvature of the laminate can be tailored without changing its mechanical properties, i.e., bending rigidity, thickness, and weight. However, curvature tailoring with initial curvature has been limited to tailoring the curvature of only one state of the two equilibrium states. In this study, we propose a curvature tailoring scheme which can tailor the curvatures of both equilibrium states by misaligning the laminate and the cylindrical tool-plate for curing. This method was verified by analysis with the Rayleigh–Ritz method and experiments. In addition, explicit equations to determine the misalignment angle and tool-plate curvature are derived for curvature tailoring of the bistable CFRP cross-ply laminates. These equations provide a simple engineering guideline for designers of bistable CFRP cross-ply laminate. The proposed curvature tailoring method gives the designer the ability to select the curvature of both equilibrium states without changing its mechanical properties, and increasing the functionality of bistable CFRP laminates.

1. Introduction

Unsymmetric lay-up sequence causes the curvature of laminate after curing due to the difference of the thermal expansion coefficient between layers. If the side length of the laminate is short, it deforms to the saddle shape, which is predicted by the classical lamination theory. If the side length of the laminate exceeds certain threshold value, however, it shows bistable phenomenon. There are two stable modes after curing the bistable CFRP laminates. Two different cylindrical shapes can be obtained by inducing external force to generate snap-through action from one equilibrium state to the other [1–3].

Rayleigh–Ritz method with von Karman nonlinearity is proposed to handle this phenomenon. Originally, Hyer [1,3] proposed the model for cross-ply laminate which is based on the polynomial displacement field and von-Karman non-linearity. This analytical modeling technique proposed by Hyer has been extended to various bistable panel problems, such as approximation of bifurcation temperature/side length [4–6], deformation behavior of angle-ply laminate [6,7], slippage effect between laminate and tool plate [8,9], identification of snap-though force [10,11], and initial curvature effect [12–14].

Bistable structure is an attractive design component for morphing structures which requires a compact, lightweight, and energy efficient shape-changing mechanism. Bistable structures do not require an energy supply to maintain their deformed shapes in their stable states. Moreover, shape transition from one stable state to another propagates automatically by residual stress if the deformation reaches a certain threshold point. Slap bracelets, self-retracting tape measures, and the Venus-flytrap robot in [15] are examples of the application of bistable structures.

The main objective of this study is the general curvature tailoring of bistable CFRP laminate using a cylindrical tool-plate. The curvature of a bistable CFRP laminate is a key feature as a design component of a morphing structure because it is closely related with efficiency and functionality of the application. The lay-up sequence is the crucial factor determining the curvature because the curvature is a function of the thermal strain difference along the thickness of the laminate. However, the lay-up sequence is closely related with the mechanical properties of laminate such as bending rigidity, thickness, and weight. Curvature tailoring schemes, which minimize the influence on these properties, is an essential research topic because these properties are important design requirements of various applications.

* Corresponding author.

http://dx.doi.org/10.1016/j.compscitech.2014.08.024
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Inducing initial curvature in bistable CFRP laminates by curing on a curved tool-plate satisfies the requirement for a curvature tailoring scheme described above because it allows tailoring of the curvature without changing the lay-up sequence of the laminate. The effect of initial curvature on bistable structures has been discussed by Ren et al. [12], Pirrera et al. [13], and Ryu et al. [14]. Ryu et al. [14] shows that the final curvature of a bistable CFRP cross-ply laminate can be easily predicted because the final curvature of the laminate with initial curvature can be expressed as the sum of the tool-plate curvature and final curvature of the laminate without initial curvature. However, initial curvature was limited to tailoring the curvature of only one state of the two equilibrium states [12–14]. Therefore, the curvature of the other state mode cannot be tailored in the bistable CFRP cross-ply laminates as shown in Fig. 1.

Inducing the initial curvature using saddle-shaped or spherical tool-plates appears to be a plausible generalization of the previous study, because both of the normal curvatures in these cases are non-zero without twisting curvature. However, putting the laminate on such tool-plates is impossible. It requires large values of extension and compression because such tool-plates are non-developable surfaces, whereas the prepreg is the developable surface. As a result, saddle or sphere shape tool-plates for curvature tailoring are impractical solutions.

A misalignment angle between the laminate and tool-plate is proposed as a general curvature tailoring scheme. Putting the laminate on a cylindrical tool-plate does not induce large values of extension and compression because the cylindrical tool-plate and flat prepreg are developable surface while the non-zero initial normal curvatures are imposed to the laminate by the misalignment angle. Detail formulation of this generalization is described in the following paragraphs.

In this paper, a curvature tailoring scheme using a misalignment angle between the tool-plate and laminate, illustrated in Fig. 2, is proposed to overcome the limitation described above. The curvature of both stable states can be tailored while minimizing the effects on other processing and design properties of the bistable CFRP laminate, such as bending rigidity, thickness and density of the lamina. Detailed formulation of the generalized initial curvature effect is presented in Section 2. The analysis by the proposed model and its verification with experiments is presented in Section 3. The simulations are focused on the cross-ply laminate in which we derive simple equation to predict final curvatures of the bistable CFRP structure because initial bending curvature can be decoupled from twisting curvature.

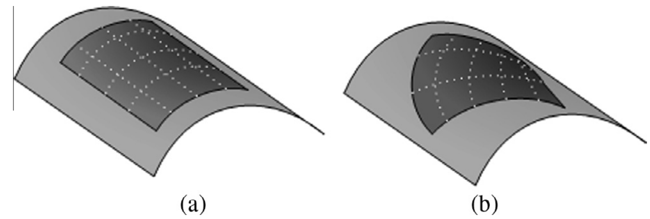

Fig. 2. Method of laying bistable CFRP laminate on a cylindrical tool-plate. (a) Fiber direction coincides with the principal curvature direction of the tool-plate. (b) Fiber direction is misaligned with the principal curvature direction of the tool-plate.

2. Analytical model development

To describe the effect of initial curvature with misalignment, the Green-Lagrangian strain field proposed in a previous study [14] must be generalized. The key feature required to describe the initial curvature effect in the previous study is the rearrangement of Green-Lagrangian strain field, which is described in Eq. (1); a flat reference state is introduced to define the final strain field and define the initial strain, i.e. a kind of inelastic strain. This concept is illustrated in Fig. 3.

$$E_{ij} = \frac{1}{2}(g_{ij} - G_{ij}) = \frac{1}{2}(g_{ij} - \delta_{ij}) - \frac{1}{2}(G_{ij} - \delta_{ij}) = E_{ij}^{fianl} - E_{ij}^{initial} \quad (1)$$

where g_{ij} is the metric at final state and G_{ij} is the metric at initial state.

A trigonometric displacement field, the basis of the Green-Lagrangian strain field for constant curvature assumption, provides more accurate results than a quadratic displacement field. [2] The Green-Lagrangian strain field to describe initial curvature without the misalignment angle between the bistable CFRP cross-ply laminate and tool-plate is described by:

$$\begin{bmatrix} E_{xx} \\ E_{yy} \\ 2E_{xy} \end{bmatrix} = \left\{ \begin{bmatrix} \xi_{xx} + \lambda_{xx}y^2 \\ \xi_{yy} + \lambda_{yy}x^2 \\ (\kappa_{xx}\kappa_{yy} + 2\lambda_{xx} + 2\lambda_{yy})xy \end{bmatrix} - \begin{bmatrix} \kappa_{xx} - \kappa_{xx}^{initial} \\ \kappa_{yy} - \kappa_{yy}^{initial} \\ 0 \end{bmatrix} z \right\} \quad (2)$$

where ξ_{xx}, ξ_{yy}, ξ_{xy}, means constant strains, λ_{xx}, λ_{yy}, means constants for quadratic normal strain variation, κ_{xx}, κ_{yy}, means principal

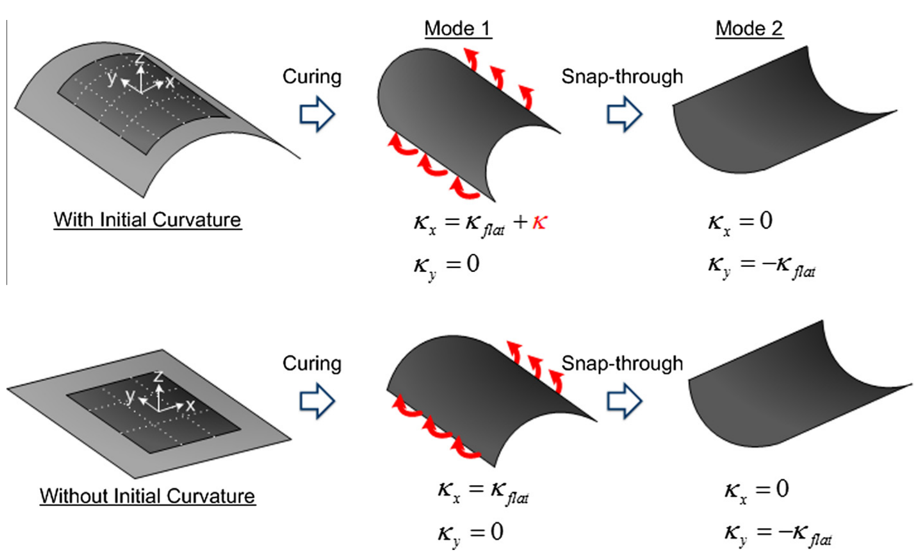

Fig. 1. Initial curvature effect with cylindrical tool-plate.