Composites Science and Technology 90 (2014) 67–73

Contents lists available at ScienceDirect

Composites Science and Technology

journal homepage: www.elsevier.com/locate/compscitech

An analytical model for predicting the stiffness and strength of pinned-joint composite laminates

A. Olmedo, C. Santiuste, E. Barbero *

Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid, Avda de la Universidad 30, 28911 Leganés, Madrid, Spain

ARTICLE INFO

Article history:
Received 4 June 2013
Received in revised form 15 October 2013
Accepted 19 October 2013
Available online 5 November 2013

Keywords:
A. Laminate
B. Strength
C. Stress concentrations
E. Welding/joining
Analytical model

ABSTRACT

An analytical model to predict bearing failure of pinned-joint composite laminates is proposed. The model combines a mass-spring model to reproduce the joint stiffness and a characteristic curve model to predict bearing damage. When bearing failure was verified at any ply, the corresponding spring element was removed from the model. The accuracy of the analytical model was validated through comparison with experimental results. Analytical model predictions agreed with the load–displacement curves and ultrasonic inspections of experimental tests. The present model predicted the different stages in the bearing failure, considering the consecutive failure of the different plies.

1. Introduction

Fastener joining is the most widely used method of assembling structural elements in the aerospace industry due to its facility to assemble, disassemble, and repair, as well as its tolerance to environmental effects [1]. Considering that the joints are very often the critical part of a structure, the soundness of their design procedure is reflected on the overall weight, performance and cost of the product. The increased stress-intensity factor at the surrounding of the hole makes the design and assembly process more critical in the case of composite joints than in those based on metallic components [2]. Structural safety needs to be ensured in the aeronautical industry, and therefore the study of mechanical joints in structural composite components has received considerable attention in both the scientific literature and aeronautical standards [3–10].

A major goal of research on the composite bolted joint has been to provide strategies to design composite mechanical joints that avoid catastrophic failure. The basic failure modes in pinned fibre reinforced materials are bearing, net-tension and shear-out failures [3]. From these failure modes only bearing damage causes progressive failure, and thus composite pinned joints are designed to fail under this mode. Bearing failure occurs in the material immediately adjacent to the contacting bolt surface due primarily to compressive stresses [4].

The complex-failure mode of composite bolted joints has been investigated by several researchers in experimental studies, e.g. [5,6]. However, due to the large range of different matrices, fibres, and lay-ups available to the designer, the use of purely empirical design procedures would be prohibitively expensive. It is essential that all the aspects of joint design are well understood, and the development of theoretical reliable models is required to optimise the prediction of composite fastener joints bearing strength.

Due to the complexity of bearing damage, several authors have proposed numerical models to predict composite bolted joint failure but few studies have focused on the development of analytical models. The bearing failure of single-lap bolted joints has been predicted with three-dimensional finite element models, showing good agreement with the experimental data [7–10]. Despite the accuracy of the finite element method [11], the development of simplified models can lead to a better understanding of this phenomenon. Analytical models include the ability to explicitly describe the physical behaviour of fastener joints, and the possibilities for conducting parametric studies. The analyses of the stress field in single-lap bolted joints have revealed that secondary bending causes non-uniform stress distributions throughout the thickness of composite laminates in the vicinity of the bolt hole [12,13], and thus the development of analytical model has been focused on the analysis of pinned joints.

Therefore the analysis of pinned joints has received considerable attention as a preliminary step in the study of composite fastener joints, but also as representative of many assembly configurations [14–16]. The Lekhnitskii method of complex stress function has been extensively used to solve the pin-loaded circular hole problem

* Corresponding author. Tel.: +34 916249965.
E-mail address: ebarbero@ing.uc3m.es (E. Barbero).

0266-3538/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compscitech.2013.10.014

in an infinite orthotropic plate. Other works, [17,18], extended this method for arbitrary load direction while considering the presence of friction. Whitworth et al. calculated the characteristic lengths in tension and compression where stresses had to be evaluated to predict bearing failure [19]. These characteristic lengths are a function of mechanical properties and stacking sequence of the laminate. In a later work, Whitworth et al. predicted the bearing failure of pin-loaded composite joints, showing conservative results when joint strength was evaluated as a function of the ratio between plate width and hole diameter [20]. Aluko and Whitworth analysed the effect of the friction coefficient on the stress distribution around the hole boundary for different staking sequences [21].

However, in these works, composite laminates were analysed as a homogeneous anisotropic single layer, and in the present work the Lekhnitskii method was applied to evaluate the failure of each ply. Thus the progressive bearing failure of different plies can be predicted. In addition, the development of a reliable analytical model to predict mechanical behaviour of pinned-joint composite laminates requires the consideration of the joint stiffness and a degradation procedure to reproduce the damage on each ply.

Mass-spring models have been widely used to reproduce the stiffness of fastener joints. Tate and Rosenfeld proposed a mass-spring model to predict the stiffness and the load distribution on bolted-joined isotropic plates [22]. Nelson et al. modified this model to analyse anisotropic composite bolted joints [23]. Recently, McCarthy et al. have developed this model to study the effect of bolt-hole clearance, friction coefficient, and torque level on multi-bolt composite joints [24,25]. Additionally, this model has been modified to predict the through-thickness stiffness in tension-loaded composite bolted joints [26]. The mass-spring models have been applied preferentially to composite bolted joints while considering the equivalent stiffness of the composite plate.

In this work, the McCarthy model was modified to include the consideration of different spring elements to reproduce the stiffness of plies with different orientation. The bearing loads and displacements determined in the mass-spring model were used to evaluate the bearing failure of each ply. When the failure of any ply was verified, the corresponding spring element was removed from the mass-spring model to reproduce the damage. In addition, an experimental test campaign was conducted to validate the analytical model predictions. Force–displacement curves and ultrasonic inspection were used to analyse the bearing failure of pinned-joints on carbon epoxy laminates.

2. Model description

An analytical model to predict bearing failure of composite laminate joints was developed using a pin configuration (Fig. 1a). A tensile load is applied to composite and auxiliary plates, and thus the bearing load is exerted by the cylindrical pin on the composite plate. This problem was modelled in two steps. First, a spring-mass model was developed to reproduce the stiffness of the joint and to calculate the bearing displacement and the bearing load applied on each ply of the laminate. Then, a two-dimensional analysis was performed to calculate the stresses on each ply and to apply a failure criterion to predict the bearing failure.

2.1. Spring-mass model

Fig. 1b illustrates the spring model used to analyse the mechanical behaviour of pinned-joint composite laminates. In this model the following simplifying considerations are assumed:

- The problem is bidimensional; no through-the-thickness variation of the parameters is considered. Therefore, the stacking sequence does not affect the laminate stiffness.
- The friction is neglected.
- The stiffness of each element in the model is lineal.

The particular model shown is for a quasi-isotropic laminate (which includes plies oriented at 0°, 90°, 45°, and −45°). However, the model can be used for any stacking sequence. The pin stiffness, $\boldsymbol{K_{pin}}$, includes the flexibility introduced by shear deformation, $\boldsymbol{K_{pin\text{-}S}}$, and bending moment, $\boldsymbol{K_{pin\text{-}B}}$. The flexibility of the composite plate under bearing loads, $\boldsymbol{K_{be}}$, was represented as three springs in parallel considering the plies with different orientation. The spring, $\boldsymbol{K_{pl}}$, represents the stiffness of the composite plate. The stiffness of the auxiliary plates is considered to be much higher than composite laminate stiffness, and thus it was not included in the model.

Once the stiffness values of all the springs shown in Fig. 2 are known, displacements can be determined for a given force. The equilibrium forces equations for each mass lead to a system of linear equations:

$$[M]\{\ddot{x}\} + [\boldsymbol{K}]\{\boldsymbol{x}\} = \{\boldsymbol{F}\}. \tag{1}$$

For quasi-static conditions the acceleration can be neglected, yielding:

$$[\boldsymbol{K}]\{\boldsymbol{x}\} = \{\boldsymbol{F}\}. \tag{2}$$

Displacement vector, $\{\boldsymbol{x}\}$, can be found by multiplying the load vector, $\{\boldsymbol{F}\}$, by the inverse of stiffness matrix $[\boldsymbol{K}]$. Considering the spring-mass model shown in Fig 2, Eq. (2) yields:

$$\begin{bmatrix} K_{pl} + (K_0 + K_{90} + K_{\pm45}) & -(K_0 + K_{90} + K_{\pm45}) & 0 \\ -(K_0 + K_{90} + K_{\pm45}) & (K_0 + K_{90} + K_{\pm45}) + \dfrac{K_{pin-S} \cdot K_{pin-B}}{K_{pin-S} + K_{pin-B}} & -\dfrac{K_{pin-S} \cdot K_{pin-B}}{K_{pin-S} + K_{pin-B}} \\ 0 & -\dfrac{K_{pin-S} \cdot K_{pin-B}}{K_{pin-S} + K_{pin-B}} & \dfrac{K_{pin-S} \cdot K_{pin-B}}{K_{pin-S} + K_{pin-B}} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F \end{Bmatrix}. \tag{3}$$

The composite plate stiffness can be found considering a composite laminate subjected to a uniform tensile load:

$$K_{pl} = \frac{\boldsymbol{E_{Lc}} \cdot \boldsymbol{W_c} \cdot \boldsymbol{t_c}}{\boldsymbol{p_c} - \boldsymbol{D}/2} \tag{4}$$

where $\boldsymbol{E_{Lc}}$ is the equivalent elasticity modulus in the longitudinal direction, which is calculated using the laminate theory; $\boldsymbol{W_c}$ and $\boldsymbol{t_c}$ are width and thickness of the composite plate, respectively; $\boldsymbol{p_c}$ is the distance between the hole surface and the plate-free end where the load is applied; and $\boldsymbol{D}$ is the hole diameter.