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Dynamic effective shear modulus of nanocomposites containing randomly distributed elliptical nano-fibers with interface effect

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ABSTRACT

Based on a surface/interface continuum theory, a theoretical study of predicting the effective dynamics properties of nanocomposites with elliptical nano-fibers is presented. The extended version of effective medium method and coherent interface model are introduced to simplify the composites with randomly distributed elliptical nano-fibers. The conformal mapping method is applied to solve the problem of typical elliptical nano-fiber. The effective elastic modulus of nanocomposites is obtained on the basis of the derived imperfect interface conditions. High accuracy of the method is demonstrated by comparing with the solutions of cylindrical nano-fibers. The effects of the shape of the nano-fibers, the material properties of interface, and the wave frequency on the dynamic shear modulus are analyzed. It is found that the interface effect is significantly related to the shape of elliptical nano-fibers. The interacting effects of the shape and the incident direction of stress waves are also examined.

1. Introduction

One-dimensional nanocomposites such as nano-wires and nano-fibers process higher tensile strength and modulus. Significant advances in the synthesis and fabrication of nanocomposites filled with nano-fibers make them perfect candidates for high-strength and light-weight multifunctional composites.

The statistical characterization of mechanical properties including the effective material properties can provide important information for the optimum design of materials [1,2]. The prediction of the effective material properties of heterogeneous composites has been the subject of considerable scientific and engineering interest [3,4]. An important character of nanocomposites is the large ratio of surface/interface area to the volume, and the surfaces or interfaces may exert great effects on the macroscopic physical and mechanical properties of nanostructures. In conventional continuum theories of elastic medium, the variation of inter atomic quantities is ignored. They cannot capture the atomic features of nanocomposites and describe the physical mechanisms of the surface/interface effects. Hence, they fail to predict the size-dependent material behavior when the characteristic size of composites scales down to the nanoscale.

The elastic properties of the interface region (between the matrix and nano-inhomogeneity), characterizing its stress–strain relationship, become very important, and should be given due consideration while formulating the overall properties of nanocomposites. Hence, an appropriate characterization of the interfaces is of significance while computing the effective properties of the material. The size-dependency of effective properties has been mostly investigated in terms of surface/interface energy. The surface/interface model proposed by Gurtin and Murdoch [5] was widely used in predicting the effective properties of nanocomposites [6–8]. Most recently, the atomistic-continuum interphase model has also been applied to account for the effect of interface to predict the effective anisotropic elastic properties of heterogeneous materials containing nano-inhomogeneities [9].

To date, most investigations on the effective material properties of nanocomposites focused on the static loadings. However, in the framework of dynamic case, the multiple scattering of elastic waves among the nano-inhomogeneities will come into being, which results in an effective medium with attenuation and dispersion. So, the dynamic effective properties caused by the multiple scattering phenomenons cannot be evaluated appropriately in the static framework. The dynamic response of nanocomposites is very important for the safely operation and optimum design of nano-structures, and it is highly desirable to establish a novel model to determine the effective characteristics in a fully dynamic framework.

Self-consistent method is an efficient way of describing the multiple scattering phenomenons of elastic waves in heterogeneous media. In the past decades, it has been successfully applied to solve the dynamic effective material properties of fiber reinforced composite materials [10–14]. Recently, the size-dependent dynamic

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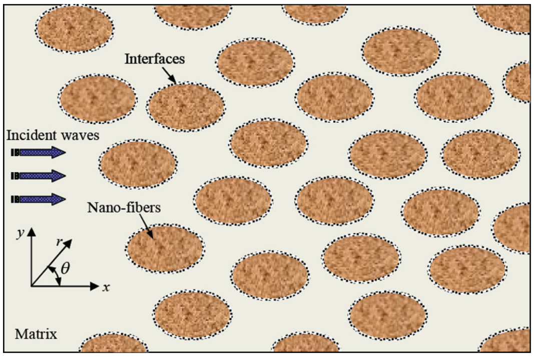

Fig. 1. An unbounded matrix with a large number of elliptical nano-fibers.

effective properties of unidirectional nanocomposites subjected to plane elastic waves including SH and SV waves have been studied by Hasheminejad and Avazmohammadi, and the interface energy effects on the effective properties were analyzed [15]. In the course of designing nanocomposites, elliptical nano-fibers are often used to obtain perfect performance of composites. The interfacial properties of these elliptical fibers on the effective material properties, however, have rarely been studied. A further optimization of the shape and interface of nano-fibers can lead to a better overall performance of composites. It is particularly useful in understanding the physical phenomena relevant to the effect of fiber shape and interface's elastic properties on wave propagation characteristics and the effective dynamic properties of nanostructured solids.

To do this, the multiple scattering of anti-plane waves from randomly distributed elliptical nano-fibers embedded in a matrix is described in this paper. By introducing the self-consistent theory, the interaction among the elliptical nano-fibers is reduced to the problem of a typical elliptical nano-fiber in effective wave field. The mapping method and an extended interface model are introduced to solve the scattering of waves around the elliptical nano-fiber. Numerical examples under different parameters are illustrated and compared with the existing solutions. The proposed model is of practical interest in ultrasonic characterization and nondestructive evaluation of nano-fiber reinforced composites with interface effects.

2. Problem formulation

An unbounded composite material with a large number of elliptical nano-fibers is considered. These identical unidirectional fibers with transversely isotropic properties are randomly distributed in the matrix, as depicted in Fig. 1. The two semi-axes of elliptical nano-fibers are a and b, respectively. The shear modulus and mass density of elliptical nano-fibers are denoted by μ_0 and $\rho_{0;}$, which are, in general, different from those (μ_m and ρ_m) of the matrix. The volume fraction of nano-fibers is n_0.

It is assumed that the composite medium is statistically homogeneous. Both the nano-fiber and matrix phases are assumed to be transversely isotropic, with the symmetry along the fiber axis (z-axis). These characters make the overall behavior of composites transversely isotropic, characterized by two effective elastic constants μ_* and ρ_*.

According to the surface/interface model in Gurtin and Murdoch [5], the interfaces around the elliptical nano-fibers are regarded as negligibly thin layer adhered to the underlying matrix material. No slipping of the layer is assumed, and its material constants are different from those of the nano-fiber and matrix. The material properties of the interface are denoted by μ_s and ρ_s.

It is supposed that an anti-plane shear wave of frequency ω with polarization parallel to the elliptical nano-fibers propagates in the composites material. Thus, the wave field within the x–y plane can be formulated in the framework of scalar wave propagation, and only the component of the displacement field in the z direction exists. Due to the multiple scattering of elastic waves among the elliptical nano-fibers, the propagating wave number and vibration amplitude will change, and the dispersion relations for shear waves will come into being. The propagating wave number is denoted as the effective wave number in effective medium. The dispersion relation about the effective wave number can be expressed as

$$\mu_*(k_*)k_*^2 - \rho_*(k_*)\omega^2 = 0, \quad (1)$$

where k_* is the effective wave number, and $\mu_*(k_*)$ and $\rho_*(k_*)$ are the effective material properties related to the effective wave number. The relation between $\mu_*(k_*)$ and $\rho_*(k_*)$ can be expressed as

$$\mu_* = \mu_m(\rho_*/\rho_m)[\mathrm{Re}(k_m/k_*)]^2, \quad (2)$$

in which $k_m = \omega\sqrt{\mu_m/\rho_m}$. For a small density contrast, it is convenient for us to suppose that the effective density is frequency-independent. So, the simple rule of mixture is used, i.e.,

$$\rho_* = n_0\rho_f + (1 - n_0)\rho_m, \quad (3)$$

where $\rho_m = c\rho_f + (1 - c)\rho_c$.

According to the hypotheses of Effective Medium Method (EMM), the interaction between lots of nano-fibers is reduced to a typical one-fiber problem. This problem is the diffraction of a monochromatic plane shear wave on an isolated nano-fiber embedded in the effective medium with the effective shear modulus and mass density.

For the anti-plane shear waves in nanocomposites, a commonly adopted procedure in micromechanics to predict the transverse shear modulus is the generalized self-consistent method (Effective Medium Method in the case of this paper). By using this method, the interaction of elastic waves among the randomly distributed nano-fibers is reduced to one elliptical nano-fiber embedded in the unknown effective medium (designated by *), as depicted in Fig. 2.

3. A typical elliptical nano-fiber in the effective medium

When the multiple scattering among the elliptical nano-fibers is simplified into the typical elliptical one-fiber problem in the effective medium, the scattering of effective waves around the typical nano-fiber comes into being. The effective wave fields around the typical nano-fiber can be described in the following forms.

To express the wave field around the typical elliptical nano-fiber by wave function, the complex variable $z = x + iy$ and its complex conjugate $\bar{z} = x - iy$ are introduced. Then the following relations can be obtained

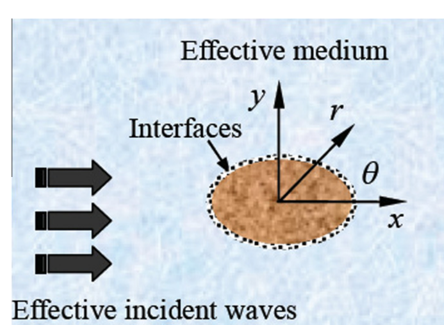

Fig. 2. The typical elliptical nano-fiber in the effective medium.