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Two non-elliptical inhomogeneities with internal uniform stresses interacting with a mode-III crack

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A B S T R A C T

We use the method of Green's functions to analyze an inverse problem in which we aim to identify the shapes of two non-elliptical elastic inhomogeneities, embedded in an infinite matrix subjected to uniform remote stress, which enclose uniform stress distributions despite their interaction with a finite mode-III crack. The problem is reduced to an equivalent Cauchy singular integral equation, which is solved numerically using the Gauss–Chebyshev integration formula. The shapes of the two inhomogeneities and the corresponding location of the crack can then be determined by identifying a conformal mapping composed in part of a real density function obtained from the solution of the aforementioned singular integral equation. Several examples are given to demonstrate the solution.

1. Introduction

The design objective of achieving uniform stress distributions inside multiple non-elliptical elastic inhomogeneities embedded in an infinite elastic matrix has attracted much attention in the literature recently (see, for example, [1–6]). The main reason for such interest lies in the fact that uniform interior stress distributions in embedded inhomogeneities are optimal in that they eliminate the possibility of stress peaks, which are well-known to cause failure of the composite containing the inhomogeneities (for example, in the manufacture of fiber-reinforced composites). In previous studies in this area, the matrix surrounding the inhomogeneities is consistently assumed to be free of any cracks. Naturally, we are led to examine the influence of a cracked matrix on the uniformity of stresses inside multiple embedded inhomogeneities. In a recent seminal study, the authors [7] have established that uniform stresses can still be maintained inside a *single* non-elliptical elastic inhomogeneity interacting with a mode-III finite crack in the matrix when the matrix is subjected to uniform remote stress.

In this work, we extend the study begun in Wang et al. [7] to the uniformity of stresses inside two non-elliptical elastic inhomogeneities interacting with a finite Griffith crack when the matrix is subjected to uniform anti-plane shear stress at infinity. We proceed as follows. By employing a Green's function from a solution derived earlier in Wang and Schiavone [4] (for a screw dislocation interacting with two elastic inhomogeneities with internal uniform stresses), we construct a conformal mapping function and a Cauchy singular integral equation, both of which contain an unknown real

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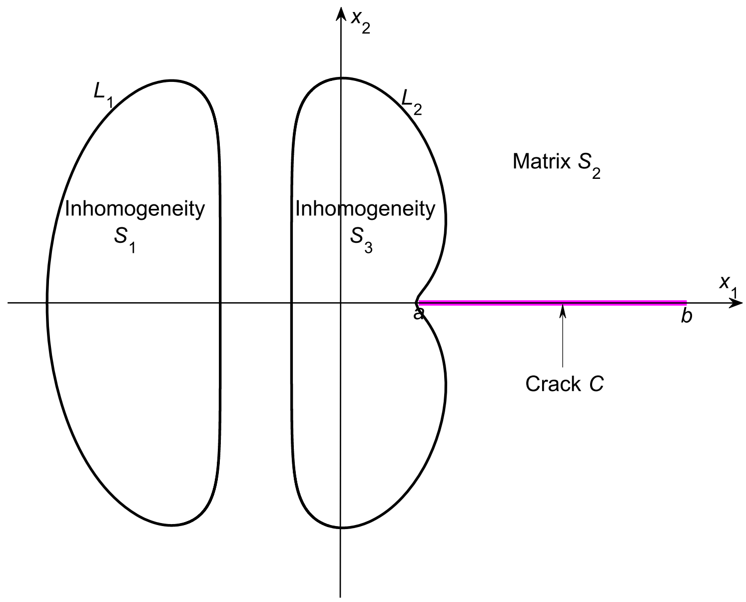

Fig. 1. Two non-elliptical elastic inhomogeneities interacting with a mode-III Griffith crack on the real axis under uniform remote stress σ_{32}^{∞}.

density function. The Cauchy singular integral equation is solved numerically for the real density function by applying the Gauss–Chebyshev integration formula [8], so that the conformal mapping function characterizing the required shapes of the inhomogeneities and the corresponding location of the matrix crack is completely determined. Our analysis indicates that the finite matrix crack plays a key role in the non-elliptical shapes of the two inhomogeneities, whereas it exerts no influence on the internal uniform stresses inside the two inhomogeneities. Several typical examples are presented to demonstrate the solution.

2. Problem formulation

Under anti-plane shear deformations of an isotropic elastic material, the two shear stress components σ_{31} and σ_{32}, the out-of-plane displacement w and the stress function ϕ can be expressed in terms of a single analytic function $f(z)$ of the complex variable $z = x_1 + \mathrm{i}x_2$ as [9]

$$\sigma_{32} + \mathrm{i}\sigma_{31} = \mu f'(z), \qquad \phi + \mathrm{i}\mu w = \mu f(z) \tag{1}$$

where μ is the shear modulus, and the two shear stress components can be expressed in terms of the stress function as [9]

$$\sigma_{32} = \phi_{,1}, \qquad \sigma_{31} = -\phi_{,2} \tag{2}$$

As shown in Fig. 1, we consider two non-elliptical elastic inhomogeneities embedded in an infinite matrix weakened by a traction-free finite Griffith crack $\{a \le x_1 \le b, x_2 = 0^{\pm}\}$ on the real axis. Let S_1, S_2, and S_3 denote the left inhomogeneity, the matrix and the right inhomogeneity, respectively, all of which are perfectly bonded through the left and the right interfaces L_1 and L_2. We denote the crack by C. The matrix is subjected to uniform remote anti-plane shear stress σ_{32}^{∞} (with $\sigma_{31}^{\infty} = 0$). In what follows, the subscripts 1, 2 and 3 are used to identify the respective quantities in S_1, S_2 and S_3. Our objective below is to determine whether uniform stresses continue to exist inside the two non-elliptical inhomogeneities when interacting with the mode-III crack.

3. The shapes of the two inhomogeneities permitting internal uniform stresses

The original interaction problem of the two inhomogeneities and the crack can be formulated as a continuous distribution of screw dislocations on the crack. The solution for a screw dislocation interacting with two non-elliptical elastic inhomogeneities permitting internal uniform stresses was recently derived by Wang and Schiavone [4]. By employing this solution as a Green's function, the following conformal mapping function for the present interaction problem can be constructed,

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