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Accounting for local capillary effects in two-phase flows with relaxed surface tension formulation in enriched finite elements

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ABSTRACT

This paper introduces a numerical method able to deal with a general bi-fluid model integrating capillary actions. The method relies first on the precise computation of the surface tension force. Considering a mathematical transformation of the surface tension virtual work, the regularity required for the solution on the evolving curved interface is weakened, and the mechanical equilibrium of the triple line can be enforced as a natural condition. Consequently, contact angles of the liquid over the solid phase result naturally from this equilibrium. Second, for an exhaustive representation of capillary actions, pressure jumps across the interface must be accounted for. A pressure enrichment strategy is used to properly compute the discontinuities in both pressure and gradient fields. The resulting method is shown to predict nicely static contact angles for some test cases, and is evaluated on complex 3D cases.

1. Introduction

Capillary-driven flows of matter are involved in a wide range of industrial and natural phenomena [1] [2] [3]. Capillary effects can be depicted as a force per unit length applied at an interface between two phases and originate from the unbalanced forces at the molecular scale on either sides of this interface [4]. Globally [5] [6] [7], three phases can be involved in capillarity: solid, liquid, and vapour phases. Due to the complexity in tracking experimentally capillary effects, a large number of numerical studies have been proposed to investigate industrial cases of capillary flows in porous and fibrous media [8] [9] [10] [11].

The present study focuses on a strategy to compute capillary effects within the context of the simulation of some manufacturing processes for high-performance composites. More especially, Liquid Resin Infusion (LRI) processes are targeted, they consist in the infiltration of a low-viscosity resin into dry fibrous preforms under a low-pressure gradient due to vacuum pulling only (<1 bar) [12] [13]. Some previous works have settled a numerical strategy to model this type of process at the macroscopic scale of the equivalent homogeneous media, *i.e.* where the fibrous preforms are not fully described across the scales [12] [14] [15]. The robustness of this approach has been demonstrated industrially [15], but some recent experi-

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mental measurements from Pucci et al. [16] have shown that capillary effects may rule the LRI processes to a great extent. Capillary pressures identified in [16] for industrial resin–fibre couples were measured up to 0.3 bar, that is to say one third of the highest pressure differential that can be employed in out-of-autoclave processes such as LRI. Besides, it is known that voids appearing during the infusion process are controlled by the competition between both capillary action and viscous dissipation at the microscopic scale [17]. Accounting for this competition at the scale of the equivalent medium would rely on identified laws and parameters that would hide the complexity of the void creation, migration, and combination [18]. In an attempt to grasp these very local phenomena, the present work aims at setting a robust numerical framework modelling of the capillary effects at the scale of the fibers.

Overall, modelling capillarity faces two main challenges related to the moving and changing liquid–vapour interface: the surface tension combines with the flow front curvature to yield the capillary driving force, and physical discontinuities must be captured across this interface. In the corresponding strongly non-linear problem, the fluid front has to accommodate to verify at the same time the fluid bulk and interfacial equilibria, plus minimise the local surface tension–curvature energy. A linearisation is then mandatory, but still, computing properly the corresponding driving force and discontinuities on the moving curved liquid–vapour interface is a considerable task that can be taken up only by combining stable numerical techniques.

First, regarding the fluid front description, a *level-set* method [19] is used here to follow and capture this interface. It has the advantage to carry a direct representation of the interface. Then, normal vectors and curvatures are explicitly known without any further reconstruction step, unlike the Volume Of Fluid (VOF) technique [20] for instance. Also, this method takes into account topological changes naturally; this is a crucial feature for the study of voids merge and split. When computing the surface tension driving force, a mathematical transformation weakens the regularity required on the interface, since the curvature no longer appears in the problem. Furthermore, the mechanical equilibrium at the junction between solid, liquid, and vapour phases, is subsequently enforced as a natural condition [21], without explicitly considering the contact angle of the liquid over the solid phase. Not prescribing the contact angle is a first step toward a dynamic contact angle representation and permits to rely only on intrinsic properties of the three phases.

The second numerical difficulty comes from the discontinuity of the stress tensor across the fluid–vapour interface. Since viscosities of both liquid and vapour are different, the normal derivatives of the velocity are also discontinuous across the interface [22], and so is the pressure according to the fluid momentum balance. Also, for the same reason, the density contrast between both fluids will induce a pressure gradient jump. Both jumps have to be properly computed since, as shown in the literature [23], errors in the pressure field lead to spurious velocities and then to a degradation of the interface [24] [25]. Various techniques have been proposed in the literature to account for these jumps [26] [27] [28]. Following [29] and Coppola-Owen et al. [30], E-FEM are used here, which consists in enlarging the space of solution for the pressure but with no change in the size of the algebraic system to be solved.

The last technical issue in capillary modelling is the coupling between the flow front motion and the flow mechanical equilibrium, *i.e.* the *level-set* problem and the Stokes equation for the fluid. A fully decoupled staggered strategy is proposed for this.

The paper is organised as follows. In Section 2, the mathematical formulation is introduced for the Stokes equations and the corresponding boundary conditions. Section 3 is devoted to the associated weak formulation that is discretised in space and time in Section 4. Section 5 sets the *level-set* method and the reparametrisation technique used. The staggered coupling approach is described in Section 6. Finally, some simulations of capillary rise of liquid compared with analytic and semi-analytic solutions are presented in Section 7. Based on the observed numerical results some conclusions are finally drawn in Section 8.

2. Mathematical formulation

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain subdivided into two sub-regions denoted $\Omega_{\mathrm{L}}(t)$ for the liquid and $\Omega_{\mathrm{V}}(t)$ for the vapour phase with $\Omega = \Omega_{\mathrm{V}}(t) \cup \Omega_{\mathrm{L}}(t)$, as shown in Fig. 1. The interface between these two fluids is denoted $\Gamma_{\mathrm{LV}}(t) = \partial\Omega_{\mathrm{V}}(t) \cap \partial\Omega_{\mathrm{L}}(t)$, also depending on time. Through the paper, the case of a liquid meniscus forming against a rigid wall, as depicted in Fig. 1, will be used as a baseline for illustrating the method and concepts. In this configuration, the rigid wall (*Solid*) is defined as a boundary of the computational domain. This boundary can be subdivided into two interfaces $\Gamma_{\mathrm{SL}} = \Omega_{\mathrm{L}} \cap \mathit{Solid}$ and $\Gamma_{\mathrm{SV}} = \Omega_{\mathrm{V}} \cap \mathit{Solid}$. Each of the three interfaces Γ_i, for $i \in \{\mathrm{SL}, \mathrm{SV}, \mathrm{LV}\}$, has a normal $\boldsymbol{n}_i$ and two tangents $\boldsymbol{t}_i^1$ and $\boldsymbol{t}_i^2$, along with one in-going tangent $\boldsymbol{T}_i$ normal to its contour (see Fig. 1). Finally, the line at the junction between the three phases (liquid, solid, vapour) is called the triple line and is denoted $L(t) = \Gamma_{\mathrm{SL}} \cap \Gamma_{\mathrm{SV}} \cap \Gamma_{\mathrm{LV}}$.

2.1. Governing equations

In the present work, every fluid is considered as Newtonian, and the corresponding stress tensor is then given by

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\mu\dot{\boldsymbol{\epsilon}}(\boldsymbol{v}), \quad \text{with} \quad \dot{\boldsymbol{\epsilon}}(\boldsymbol{v}) = \frac{1}{2}\left(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^{\mathsf{T}}\right) \tag{1}$$

with $\boldsymbol{v}$ the velocity, p the pressure and μ the constant viscosity of the fluid (*i.e.* $\mu = \mu_L$, $\forall \boldsymbol{x} \in \Omega_{\mathrm{L}}$ and $\mu = \mu_V$, $\forall \boldsymbol{x} \in \Omega_{\mathrm{V}}$). Assuming that the fluids are incompressible, both momentum balance and mass conservation equations yield the Stokes equations:

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