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Comparison of stochastic and interval methods for uncertainty quantification of metal forming processes

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A B S T R A C T

Various sources of uncertainty can arise in metal forming processes, or their numerical simulation, or both, such as uncertainty in material behavior, process conditions, and geometry. Methods from the domain of uncertainty quantification can help assess the impact of such uncertainty on metal forming processes and their numerical simulation, and they can thus help improve robustness and predictive accuracy. In this paper, we compare stochastic methods and interval methods, two classes of methods receiving broad attention in the domain of uncertainty quantification, through their application to a numerical simulation of a sheet metal forming process.

1. Introduction

Various sources of uncertainty can arise in metal forming processes, or their numerical simulation, or both, such as uncertainty in material behavior, uncertainty in process conditions including friction properties, and uncertainty in geometrical properties. Here, uncertainty can refer to manufacturing variability in material behavior, process conditions, and geometry, or it can refer to the imperfect representation or incomplete knowledge of these properties in a numerical simulation. The presence of such sources of uncertainty can raise the challenge of taking into account such uncertainty in the design, the control, the optimization, the maintenance, and so forth of metal forming processes, as well as in their numerical simulation.

In the domains of uncertainty quantification and computational mechanics, new methods for the analysis and management of uncertainty are under development, see, for instance, [1–12]. These developments are very rich, and two classes of new methods receiving broad attention are the stochastic methods and the interval methods. On the one hand, stochastic methods represent uncertainty by means of probability distributions, and they rely on the probability theory to determine the impact of sources of uncertainty on quantities that depend on them. On the other hand, interval methods represent uncertainty by means of intervals, and they rely on interval arithmetic, or optimization theory, or both to determine the impact of sources of uncertainty on quantities that depend on them. Around these core tasks of representing uncertainty and determining the impact of sources of uncertainty on quantities that depend on them, research in uncertainty quantification and computational mechanics builds new methods for accounting for uncertainty in design, control, optimization, maintenance, and many other engineering tasks. These new methods from the domains of uncertainty quantification and computational mechanics can be usefully applied to the analysis and management of uncertainty in metal forming processes and their numerical simulation.

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In this paper, we compare the stochastic and interval methods through their application to a numerical simulation of a sheet metal forming process. Our intent is to provide some insight into how stochastic and interval methods handle the core tasks of representing uncertainty and determining the impact of sources of uncertainty on quantities that depend on them. The integration of these core tasks in new methods for design, control, optimization, maintenance, and other engineering tasks under uncertainty is beyond the scope of this paper. Further, we note that whereas we had already applied in two previous papers [9,13] stochastic methods to a metal forming problem with uncertain material properties, we apply here stochastic and interval methods to a metal forming problem involving not only uncertain material properties but also uncertain friction and geometrical characteristics.

This paper is organized as follows. First, in Secs. 2 and 3, we provide concise overviews of the stochastic and interval methods. Then, in Sec. 4, the core of this paper, we compare them in the context of the quantification of uncertainty in a numerical simulation of a sheet metal forming process.

2. Stochastic methods

Let us assume that we consider a mechanical problem that lends itself well to a representation in terms of a transformation of input parameters into a quantity of interest. Specifically, let us assume that there are a finite number, say d, of vector-valued input parameters, which we denote by $\boldsymbol{x}^1, \ldots, \boldsymbol{x}^d$, with $\boldsymbol{x}^1 = (x_1^1, \ldots, x_{s_1}^1)$ of dimension s_1 up to $\boldsymbol{x}^d = (x_1^d, \ldots, x_{s_d}^d)$ of dimension s_d, that are transformed through a function, which we denote by f, into a scalar quantity of interest, which we denote by y:

$$y = f(\boldsymbol{x}^1, \ldots, \boldsymbol{x}^d) \tag{1}$$

please note that in these expressions, the superscripts serve to index the vector-valued input parameters. For example, in a mechanical problem involving a metal forming process, one of the vector-valued input parameters, say $\boldsymbol{x}^1$, could collect material properties, another vector-valued input parameter, say $\boldsymbol{x}^2$, could collect friction properties, another vector-valued input parameter, say $\boldsymbol{x}^3$, could collect geometrical properties, and so forth; the quantity of interest y could represent a property of the deformed piece such as a magnitude of a residual stress or a displacement component at a certain location; and the function f could represent how this quantity of interest depends on these vector-valued input parameters in this metal forming process or a numerical simulation of it.

Let us assume that the vector-valued input parameters are uncertain. Within this context, we discuss below some of the key concepts of how stochastic methods allow the uncertainty in the vector-valued input parameters to be represented, its impact on the quantity of interest to be determined, and a sensitivity analysis to be carried out.

We note that this section provides only a concise overview; we refer the reader to [2,4,5,7–12] and references therein for more comprehensive texts. Further, we note that although we consider for the sake of conciseness a context involving uncertain scalars and vectors, stochastic methods are not limited to uncertain scalars and vectors and can readily deal with uncertain matrices, fields, functions, operators, and other quantities.

2.1. Characterization of uncertainty

Stochastic methods account for sources of uncertainty in a mechanical problem by representing them by using probability distributions. As such, the application of stochastic methods typically begins with identifying suitable probability distributions for these sources of uncertainty from available information, a task called the characterization of uncertainty.

In the present context, stochastic methods entail the representation of the uncertain vector-valued input parameters by (vector-valued) random variables, which we denote by $\boldsymbol{X}^1 = (X_1^1, \ldots, X_{s_1}^1)$ up to $\boldsymbol{X}^d = (X_1^d, \ldots, X_{s_d}^d)$; please note that it is customary in the probability theory [14] to denote random variables by using uppercase letters. We denote their probability distributions by $\pi_{\boldsymbol{X}^1} = \pi_{(X_1^1, \ldots, X_{s_1}^1)}$ up to $\pi_{\boldsymbol{X}^d} = \pi_{(X_1^d, \ldots, X_{s_d}^d)}$, respectively:

$$\boldsymbol{X}^1 \sim \pi_{\boldsymbol{X}^1}, \quad \ldots \quad, \boldsymbol{X}^d \sim \pi_{\boldsymbol{X}^d} \tag{2}$$

here, with $1 \le j \le d$, we denote by $\boldsymbol{X}^j \sim \pi_{\boldsymbol{X}^j}$ that $\boldsymbol{X}^j$ is distributed according to $\pi_{\boldsymbol{X}^j}$, by which the probability theory understands that $\pi_{\boldsymbol{X}^j}$ is a function that assigns to any meaningful subset $\mathcal{B}^j$ of $\mathbb{R}^{s_j}$ the probability $\pi_{\boldsymbol{X}^j}(\mathcal{B}^j)$ that the value taken by $\boldsymbol{X}^j$ is in $\mathcal{B}^j$. From the mechanical point of view, if the uncertainty refers to manufacturing variability, the probability distributions $\pi_{\boldsymbol{X}^1}, \ldots, \pi_{\boldsymbol{X}^d}$ can be interpreted as describing frequencies of occurrence of values of the uncertain vector-valued input parameters; and if the uncertainty refers to an imperfect representation or incomplete knowledge, they can be interpreted as describing plausabilities of values of the uncertain vector-valued input parameters.

We assume that the partitioning of the input uncertainty into the uncertain vector-valued input parameters is such that these uncertain vector-valued input parameters are represented appropriately by mutually statistically independent random variables, by which the probability theory understands that the joint probability distribution $\pi_{(\boldsymbol{X}^1, \ldots, \boldsymbol{X}^d)}$ of $\boldsymbol{X}^1, \ldots, \boldsymbol{X}^d$ is the product of the probability distributions $\pi_{\boldsymbol{X}^1}, \ldots, \pi_{\boldsymbol{X}^d}$:

$$\pi_{(\boldsymbol{X}^1, \ldots, \boldsymbol{X}^d)} = \pi_{\boldsymbol{X}^1} \times \ldots \times \pi_{\boldsymbol{X}^d} \tag{3}$$