C. R. Mecanique ••• (••••) •••–•••

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Model reduction, data-based and advanced discretization in computational mechanics

A door to model reduction in high-dimensional parameter space

Vers des modèles réduits dans les espaces de très grande dimension

Charles Paillet *, David Néron, Pierre Ladevèze

LMT (ENS Paris-Saclay, CNRS, Université Paris-Saclay), 61, avenue du Président-Wilson, 94235 Cachan cedex, France

ARTICLE INFO

Article history:
Received 12 December 2017
Accepted after revision 2 April 2018
Available online xxxx

Keywords:
Reduced Order Model (ROM)
PGD
Multiparametric

Mots-clés :
Réduction de modèles
PGD
Multiparamétrique

ABSTRACT

Model reduction techniques such as Proper Generalized Decomposition (PGD) are decision-making tools that are about to revolutionize many domains. Unfortunately, their computation is still problematic for problems involving many parameters, for which one has to face the "curse of dimensionality". An answer to this challenge is given in solid mechanics by the so-called "parameter-multiscale PGD", which is based on Saint-Venant's principle. In this article, a model problem composed of up to a thousand parameters is presented, showing that the method is able to overcome the "curse of dimensionality".

RÉSUMÉ

Les modèles réduits, en particulier ceux basés sur la *Proper Generalized Decomposition* (PGD) sont des outils de conception qui s'apprêtent à révolutionner la simulation numérique. Malheureusement, pour les problèmes à grand nombre de paramètres, la « malédiction de la dimensionnalité » semble être une limitation majeure. Nous proposons, avec la *parameter-multiscale PGD*, une solution à ce problème basée sur le principe de Saint-Venant. Un cas test comprenant jusqu'à mille paramètres est présenté dans cet article et prouve que la méthode permet bien de s'affranchir de la « malédiction de la dimensionnalité ».

1. Introduction

Numerical simulation has made a forceful entry into design and analysis offices. This revolution, which is anything but complete, has entered a new stage, called simulation-driven "robust" design, and leads to a major scientific challenge: simulations should be performed in quasi real time. The key is a new generation of reduced-order methods that comprises essentially the Proper Orthogonal Decomposition (POD), the Proper Generalized Decomposition (PGD), and the Reduced-Basis Method (RB), the basics and recent developments of which are given in [1]. Problems that must be solved may involve

* Corresponding author.
E-mail addresses: paillet@lmt.ens-cachan.fr (C. Paillet), neron@lmt.ens-cachan.fr (D. Néron), ladeveze@lmt.ens-cachan.fr (P. Ladevèze).

https://doi.org/10.1016/j.crme.2018.04.009
1631-0721/

Please cite this article in press as: C. Paillet et al., A door to model reduction in high-dimensional parameter space, C. R. Mecanique (2018), https://doi.org/10.1016/j.crme.2018.04.009

a very high number of degrees of freedom, with multiple scales or interactions between several physics and can be associated with variable or uncertain parameters. Model reduction methods, together with the notions of offline and online calculations, also open the way to new approaches where simulation and analysis of structures can be carried out almost in real time.

This work is based on the PGD, which was introduced in [2,3] for the treatment of nonlinear time-dependent problems in solid mechanics. Many developments have been made over the last thirty years: multiscale, multiphysics, stochastic or non-stochastic parameters, acoustics, large displacements and deformations... In [4], the interested reader can find a synthesis of most of the developments carried out in Cachan, where the LATIN method plays a central role. A number of tools are now mature and have been applied to industrial cases and then, are competitors of classical computational methods (see book [5]). The PGD not only makes it possible to construct reduced models that can be used in real time, but it also reduces drastically the whole calculation time in many situations. However, a major limitation is still the number of parameters that can be involved (no more than a dozen, as it will be discussed in the following). This paper starts with a brief description of the classical PGD, highlighting its limitations for problems of high dimension.

Several attempts have been made to solve problems with a large number of parameters. For example, enhancements can be introduced by iterative solvers with conditioner [6] or more complex approximations of the data structure. The PGD uses the so-called separated variable representation, or canonical decomposition, but other compressed high dimensional field descriptions has been introduced: Tucker tensors, Tensor Train format or Hierarchical Tucker format [7–11]. However, the generic formulation of those tools can be a limitation for our applications.

On the contrary, we develop in this paper a physically based approximation introduced in [12,13]. Our proposal, named "parameter-multiscale" PGD is built on the Saint-Venant Principle, which works for many models in Physics. This "principle" highlights two different levels of parametric influence, which drives us to introduce a multiscale description of the parameters and to separate a "macro" and a "micro" scale, as it is classically done for space or time [14]. To implement this vision, a completely discontinuous spatial approximation is needed. Thus, we use the Weak-Trefftz Discontinuous Method introduced in [15] and applied in [16] for the calculation of "medium frequency" phenomena. In this paper, we first recall the basics of the parameter-multiscale PGD. Then, new developments are introduced, the main one being the computation of the algorithm on a 3D problem up to the second iteration, which leads to very small errors. This is done for problems with more than a thousand parameters, which shows that the method is able to overcome the "curse of dimensionality". Additional results put forward the capability and the limits of the parameter-multiscale PGD for solving problems with numerous parameters.

2. Model problem

One considers an elastic media that occupies the domain $\Omega \subset \mathbb{R}^3$ divided into N subdomains or elements Ω_E, $E \in \mathbf{E}$. The parameter μ_E is associated with the rigidity of Ω_E and, for the sake of simplicity, we consider that μ_E is a scalar belonging to $[-1/2; 1/2]$. Let us introduce $\underline{\mu} \equiv \{\mu_E\}_{E\in\mathbf{E}}$, the corresponding space being $\mathbf{\Sigma}_\mu$. Note that, for an isentropic material, the maximum number of independent parameters per subdomain or element is 2. After discretization, any spatial field is approximated by N degrees of freedom, and the problem to solve can be written as:

$$\begin{aligned} &\text{Find } \underline{X}(\underline{\mu}) \in \boldsymbol{X} \text{ where } \boldsymbol{X} : \begin{cases} \mathbf{\Sigma}_\mu & \rightarrow \ \boldsymbol{U} = \mathbb{R}^N \\ \underline{\mu} & \rightarrow \ \underline{X}(\underline{\mu}) \end{cases} \text{ such that:} \\ &\forall \underline{\mu} \in \mathbf{\Sigma}_\mu \qquad \boldsymbol{A}(\underline{\mu})\underline{X}(\underline{\mu}) = \underline{F}_d(\underline{\mu}) \end{aligned} \tag{1}$$

where $\boldsymbol{A}$ is a linear positive definite operator depending on $\underline{\mu}$. $\underline{F}_d$ is a given loading that could also depend on $\underline{\mu}$, but which will be considered constant for the sake of simplicity.

Fig. 1 shows an illustration of the model problem. This is a cube submitted to a uniaxial traction displacement, the opposite face sliding freely along a fixed surface. The parameters $\underline{\mu}$ are proportional to the local Young modulus and could be interpreted as damage intensity.

3. The standard PGD

The standard parameter PGD is described in [17]. For practical details, the reader can refer to [1]. In this method, the "curse of dimensionality" is bypassed using a separated variable representation (2):

$$\underline{X}(\underline{\mu}) = \underline{X}(\mu_1, \ldots, \mu_N) \approx \underline{X}^M(\mu_1, \ldots, \mu_N) = \sum_{j=1}^{M} \underline{\widetilde{X}}^j \prod_{E\in\mathbf{E}} g_E^j(\mu_E) \tag{2}$$

where M is the number of modes of the approximation, $\underline{\widetilde{X}}^j$ a spatial vector and the g_E^j are functions of only one parameter μ_E.

At each iteration, a greedy procedure enriches the solution with one new mode, which is computed using a fixed-point algorithm, for example. The convergence of the method is illustrated in Fig. 2 for three different test cases, with respectively 8, 27 (as in Fig. 1), and 64 parameters, using a normalized residual as an error indicator:

Please cite this article in press as: C. Paillet et al., A door to model reduction in high-dimensional parameter space, C. R. Mecanique (2018), https://doi.org/10.1016/j.crme.2018.04.009