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A coated rigid elliptical inclusion loaded by a couple in the presence of uniform interfacial and hoop stresses

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A B S T R A C T

We consider a confocally coated rigid elliptical inclusion, loaded by a couple and introduced into a remote uniform stress field. We show that uniform interfacial and hoop stresses along the inclusion–coating interface can be achieved when the two remote normal stresses and the remote shear stress each satisfy certain conditions. Our analysis indicates that: (i) the uniform interfacial tangential stress depends only on the area of the inclusion and the moment of the couple; (ii) the rigid-body rotation of the rigid inclusion depends only on the area of the inclusion, the coating thickness, the shear moduli of the composite and the moment of the couple; (iii) for given remote normal stresses and material parameters, the coating thickness and the aspect ratio of the inclusion are required to satisfy a particular relationship; (iv) for prescribed remote shear stress, moment and given material parameters, the coating thickness, the size and aspect ratio of the inclusion are also related. Finally, a harmonic rigid inclusion emerges as a special case if the coating and the matrix have identical elastic properties.

1. Introduction

It is well known that a uniform distribution of interfacial normal and tangential stresses is optimal in that it will eliminate any stress peaks at the interface between an inclusion and the surrounding material [1]. Furthermore, a uniform distribution of hoop stress along the edge of a hole or inclusion is also known to be ideal in the design of what Cherepanov refers to as "equally strong outlines of holes" [2]. The design of inclusions with constant interfacial and hoop stresses has attracted considerable attention in the literature (see, for example, [1,3–7]). In particular, confocal elliptical interfaces can be used to achieve the design objective of uniform interfacial and hoop stresses for a three-phase elliptical inclusion [1]. In all these previous investigations, the inclusion itself is free of any external loading. It is of great practical and theoretical interest to ask whether, if a rigid inclusion were loaded, for example, by a couple, it would still be possible to achieve uniform interfacial and hoop stresses along the inclusion boundary. This question forms the basis of the present study.

In this paper, we consider a rigid elliptical inclusion, loaded by a couple moment and bonded to an infinite elastic matrix through a coating consisting of two confocal elliptical interfaces. We develop one condition on remote normal stresses and another on remote shear stress that ensure that the interfacial and hoop stresses along the inclusion–coating interface are

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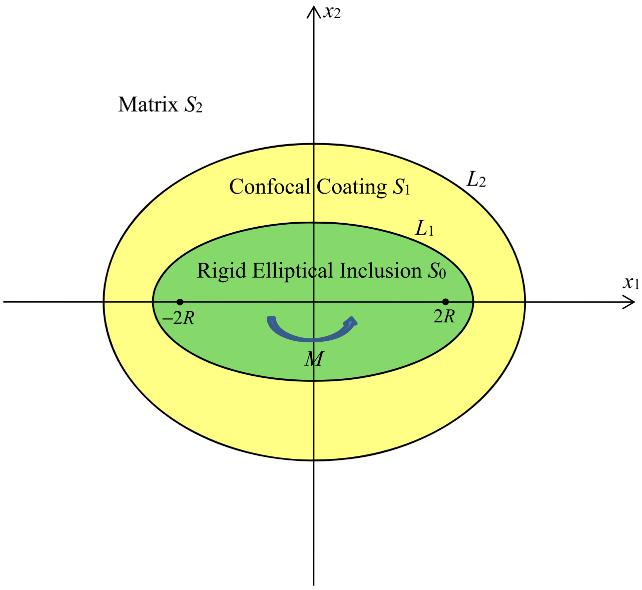

Fig. 1. A confocally coated rigid elliptical inclusion loaded by a couple.

uniformly distributed. For prescribed remote normal stresses and material parameters, a relationship is established between the coating thickness and the aspect ratio of the inclusion. In the case of prescribed remote shear stress, couple moment and material parameters, we establish a relationship among the coating thickness, the size, and the aspect ratio of the inclusion.

2. Complex variable formulation

In a Cartesian coordinate system $Ox_1x_2x_3$, in the case of plane elastostatics, the stresses $(\sigma_{11}, \sigma_{22}, \sigma_{12})$, displacement components (u_1, u_2) and the stress functions (ϕ_1, ϕ_2) can be expressed in terms of two analytic functions $\varphi(z)$ and $\psi(z)$ of the complex variable $z = x_1 + \mathrm{i}x_2$ as [8]

$$\begin{aligned} &\sigma_{11} + \sigma_{22} = 2\left[\varphi'(z) + \overline{\varphi'(z)}\right], \\ &\sigma_{22} - \sigma_{11} + 2\mathrm{i}\sigma_{12} = 2\left[\bar{z}\varphi''(z) + \psi'(z)\right], \end{aligned} \tag{1}$$

$$\begin{aligned} &2\mu(u_1 + \mathrm{i}u_2) = \kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} \\ &\phi_1 + \mathrm{i}\phi_2 = \mathrm{i}\left[\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)}\right] \end{aligned} \tag{2}$$

where $\kappa = 3 - 4\nu$ for plane strain, $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress, μ and ν $(0 \le \nu \le 1/2)$ are the shear modulus and Poisson's ratio, respectively. In addition, the stresses are related to the stress functions through [9]

$$\begin{aligned} &\sigma_{11} = -\phi_{1,2}, \qquad \sigma_{12} = \phi_{1,1} \\ &\sigma_{21} = -\phi_{2,2}, \qquad \sigma_{22} = \phi_{2,1} \end{aligned} \tag{3}$$

3. General solution

Consider a rigid elliptical inclusion bonded to an infinite elastic matrix through a confocal coating. Let S_0, S_1 and S_2 denote the rigid inclusion, the coating and the matrix, respectively, all of which are perfectly bonded across two confocal elliptical interfaces L_1 and L_2, the common foci of which are located at $z = \pm 2R$ $(R > 0)$ on the real axis, as shown in Fig. 1. The rigid inclusion is loaded by a couple of moment M and the matrix is subjected to remote uniform in-plane stresses $(\sigma_{11}^\infty, \sigma_{22}^\infty, \sigma_{12}^\infty)$. In what follows, the subscripts 1 and 2 will be used to identify the respective quantities in S_1 and S_2.

We first introduce the following conformal mapping function [8]

$$z = \omega(\xi) = R\left(\xi + \frac{1}{\xi}\right), \qquad \xi = \omega^{-1}(z) = \frac{z}{2R} + \sqrt{\frac{z^2}{4R^2} - 1} \tag{4}$$

which maps the segment $[-2R, 2R]$ onto the unit circle in the ξ-plane and the two interfaces L_1 and L_2 onto two coaxial circles with radii R_1 and R_2. Thus S_1 and S_2 are mapped onto $R_1 \le |\xi| \le R_2$ and $|\xi| \ge R_2$, respectively. For the sake of convenience and without loss of generality, we write $\varphi_i(\xi) = \varphi_i(\omega(\xi))$, $\psi_i(\xi) = \psi_i(\omega(\xi))$, $i = 1, 2$.