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Unsteady free surface flow in porous media: One-dimensional model equations including vertical effects and seepage face

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ABSTRACT

This note examines the two-dimensional unsteady isothermal free surface flow of an incompressible fluid in a non-deformable, homogeneous, isotropic, and saturated porous medium (with zero recharge and neglecting capillary effects). Coupling a Boussinesq-type model for nonlinear water waves with Darcy's law, the two-dimensional flow problem is solved using one-dimensional model equations including vertical effects and seepage face. In order to take into account the seepage face development, the system equations (given by the continuity and momentum equations) are completed by an integral relation (deduced from the Cauchy theorem). After testing the model against data sets available in the literature, some numerical simulations, concerning the unsteady flow through a rectangular dam (with an impermeable horizontal bottom), are presented and discussed.

1. Introduction

The Boussinesq equation [1–4] is the prototype of one-dimensional model equations for the study of unsteady free surface flow in porous media. As is well known, this equation can be obtained by coupling the Dupuit approximation with the Darcy law. To investigate how the flow is affected by vertical velocity, the Boussinesq equation is often replaced by a series of extended Boussinesq equations [5–12]. These equations are used extensively to describe the water waves propagation in porous media as a consequence of tide-induced fluctuations [5–10] and wave interactions with porous structures [7,11,12]. A problem arising from the use of these extended Boussinesq equations is that the seepage face development and the capillary fringe effects are neglected. Whereas a series of alternative Boussinesq equations are proposed to evaluate the capillary fringe effects [13–15], how to evaluate the seepage face development with a one-dimensional model is a problem still not completely solved [16,17].

An attempt to determine the height of the seepage face with the Boussinesq equation is performed in [16] by incorporating in the model equation a new equation. However, this equation, expressed by an inequality, "*is an intuitive approximation of how a seepage face could form assuming Dupuit–Forchheimer flow*" [16]. In the light of this statement, the use of this inequality in conjunction with the Boussinesq equation must be considered as a mathematical artifice that is used to force fit the model predictions to experimental evidences. This kind of approaches, based on mathematical models of dubious physical meaning, is commonly used also in other field (e.g., open channel flow and pipe flow – a critical review can be found in [18–25]), and determines a dangerous separation between the obtained solution and the problem under investigation.

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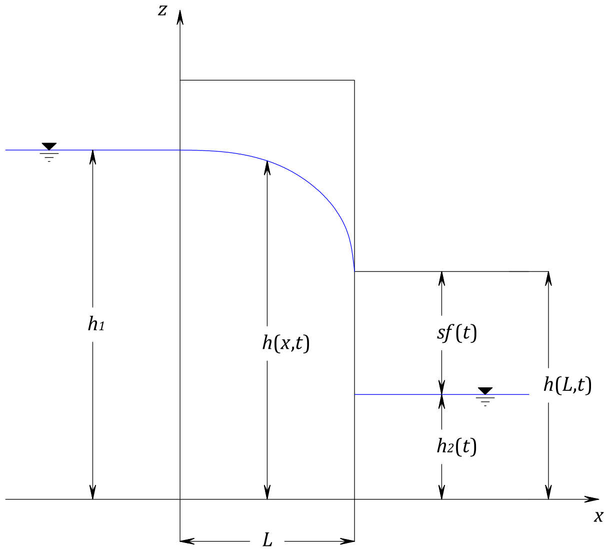

Fig. 1. Definition sketch.

In steady-state flow conditions, the seepage face height is evaluated analytically in a simple configuration (seepage flow in a homogeneous rectangular dam) by Polubarinova-Kochina [26]. This analytical solution, given in the elliptic integral form of the first kind, is computed numerically in [27]. For some simple configurations, approximate methods to compute the seepage face height can be found in [28–33].

In the field of numerical models, the seepage face problem is analyzed, among others, in [34–42].

Within the framework of extended Boussinesq equations (in the above-mentioned meaning), this note proposes a new set of one-dimensional model equations including vertical effects and seepage face. The model is built considering the archetypal case of the two-dimensional unsteady isothermal flow of an incompressible fluid in a non-deformable, homogeneous, isotropic, and saturated rectangular dam (with an impermeable horizontal bottom and zero recharge). The capillary effects are neglected. In order to take into account the seepage face development, the model equations (obtained by coupling a Boussinesq-type model for nonlinear water waves with Darcy's law) are completed by an integral relation (deduced from the Cauchy theorem). After testing the model against data sets available in the literature, some numerical simulations are presented and discussed.

The note is structured as follows: Section 1 presents the problem formulation; Section 2 is devoted to the one-dimensional model equations; the details of numerical simulations and discussions of the results are provided in Section 3; conclusions are given in Section 4. In order to make reading easier, some mathematical considerations are included in the Appendixes at the end of the paper.

2. Flow problem and governing equations

The free boundary problem under consideration concerns the two-dimensional unsteady isothermal free surface flow of an incompressible fluid (water) in a non-deformable, homogeneous, isotropic and saturated rectangular dam of width L, with an impermeable horizontal bottom. Fig. 1 shows the geometric setup: the horizontal x-axis and the vertical z-axis form a Cartesian coordinate system; t is the time variable; $h_1 =$ constant and $h_2(t)$ are, respectively, the water levels at $x = 0$ and $x = L$; $h(x,t)$ is the free surface elevation; $sf(t) = h(L,t) - h_2(t)$ is the seepage face height.

With zero recharge and neglecting the capillary effects, the free surface is a sharp interface between saturated and dry zones. The flow problem consists in finding the evolution of the free surface $h(x,t)$, the seepage velocity field $\boldsymbol{v}(x,z,t) = (v_x, v_z)$ (where $v_x(x,z,t)$ and $v_z(x,z,t)$ are the horizontal and vertical seepage velocity components, respectively) and the pressure field $p(x,z,t)$ in the unbounded flow domain $\Omega = [0, L] \times [0, h]$ for any $t > 0$.

The problem is governed by the continuity and momentum equations with appropriate boundary and initial conditions. In the range of validity of Darcy's hypothesis [43], the system equations are given as [2,43]:

$$\nabla \cdot \boldsymbol{v} = 0 \quad \text{in } \Omega \text{ (continuity equation)} \tag{1}$$

$$\boldsymbol{v} = -K\nabla\left(z + \frac{p}{\rho g}\right) \quad \text{in } \Omega \text{ (momentum equation)} \tag{2}$$

where K is the constant hydraulic conductivity, ρ the constant water density, and g the gravitational acceleration. It is important to note that the assumption of non-deformable porous medium implies that the storativity vanishes.