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Stability analysis of internally damped rotating composite shafts using a finite element formulation

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ABSTRACT

This paper deals with the stability analysis of internally damped rotating composite shafts. An Euler–Bernoulli shaft finite element formulation based on Equivalent Single Layer Theory (ESLT), including the hysteretic internal damping of composite material and transverse shear effects, is introduced and then used to evaluate the influence of various parameters: stacking sequences, fiber orientations and bearing properties on natural frequencies, critical speeds, and instability thresholds. The obtained results are compared with those available in the literature using different theories. The agreement in the obtained results show that the developed Euler–Bernoulli finite element based on ESLT including hysteretic internal damping and shear transverse effects can be effectively used for the stability analysis of internally damped rotating composite shafts. Furthermore, the results revealed that rotor stability is sensitive to the laminate parameters and to the properties of the bearings.

1. Introduction

A precise prediction of damping effects is basically necessary in the stability analysis of rotor dynamic behaviour. Damping is considered as an internal damping such as material damping or as an external damping as in the case of bearing damping, and it is principally modeled using viscous or hysteretic damping. The basic difference between viscous and hysteric models is that the dissipation of energy by viscous damping depends on frequency, while the dissipation of energy by hysteretic damping does not. In composite rotor dynamic field, internal damping can be meaningful because of the damping capacity of the matrix [1]. Moreover, most materials, such as metallic materials, carbon/epoxy materials, and viscoelastic materials show a vibratory damping behaviour that looks like hysteretic internal damping much more than like viscous internal damping [2]. Due to the specific strength and stiffness of the fiber-reinforced composite materials, metal shafts have been replaced by composite shafts in many applications, such as drive shafts for helicopters and automotive industries [3–7]. These materials offer benefits in terms of reduction of the weight and augmentation of the strength, stiffness and damping capacity, and provide structural designers the possibility of obtaining required behaviours by changing the stacking sequence of the composite layers in terms of number and orientation of layers [8,9].

Many researchers studied the damping effects on rotor dynamic and stability behaviour [1,2,10–15]. First investigations by Newkirk [10] showed that rotors may undergo violent whirling at speeds above the first critical speed because of in-

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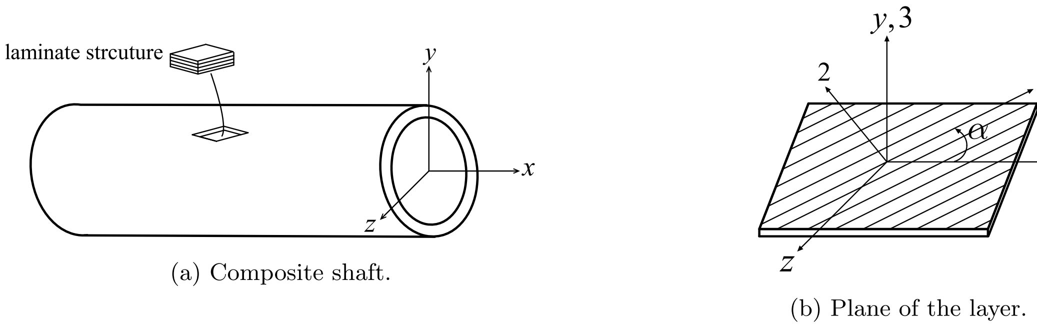

Fig. 1. Composite shaft.

ternal damping. After that, Genta [12] explained similarly that the hysteretic internal damping of the rotating elements of a structure is stabilizing in the subcritical range and destabilizing in the supercritical range; they proved that an error is made when admitting that hysteretic internal damping of rotating elements is destabilizing at all rotational speeds. Montagnier and Hochard [2] demonstrated likewise that damping associated with the non-rotating elements of the structure has a stabilizing effect, while damping associated with the rotating elements may provoke instability in the supercritical range. Besides, many researchers have studied the combined influence of internal and external damping. The results show that rotor stability is enhanced by increasing bearing damping; however, increasing internal damping can reduce the instability threshold [11,16]. Pereira and Silveira [11] used an optimization techniques to avoid the instability, reduce the unbalanced response, and augment the stability limit speed. They adopted the Equivalent Modulus Beam Theory (EMBT) developed by Singh and Gupta [17,18]. In fact, EMBT has many limitations and is only valid for symmetric stacking sequences. Simplified Homogenized Beam Theory (SHBT) based on Timoshenko's beam theory has been developed by Sino et al. [14] to consider the effects of the stacking sequence and internal damping of composite material. Jacquet-Richardet et al. [15] illustrated and validated the theoretical formulation proposed by Sino et al. [14] by developing an experimental setup to analyse the dynamic instability of an internally damped rotating composite shaft. More recently, Ben Arab et al. [19] developed ESLT based on Timoshenko's beam theory to consider the effects of stacking sequence, fiber orientations, and shear-normal coupling. ESLT considers the laminated shaft made of several orthotropic layers as an equivalent single layer having mechanical properties equivalent to those of all the layers. The authors proved that ESLT is quite adequate for the dynamic analysis of rotating composite shafts in both symmetric and non-symmetric stacking sequences.

In this paper, an Euler–Bernoulli shaft finite element formulation based on Equivalent Single Layer Theory (ESLT) is adopted. ESLT consists in considering a laminated shaft made of several orthotropic layers as an equivalent single layer having mechanical properties equivalent to those of all the layers [19,20]. The developed formulation considers the translatory, rotary inertia, and gyroscopic effects as well as the shear transverse effect that was introduced in the shape functions. This formulation is developed to analyse the effects of hysteric internal damping, fiber orientation, and stacking sequence on natural frequencies, critical speeds, and instability thresholds of internally damped rotating composite shafts. The technical contribution of the present investigation is to study the stability behaviour of internally damped rotating composite shaft considering the shear transverse effect by developing an Euler–Bernoulli shaft finite element formulation based on Equivalent Single Layer Theory (ESLT). A brief description of the theoretical background concerning the hysteretic internal damping modeling is presented. First, the kinetic energy T and the deformation energy Π of the rotor system are established. Then, the finite element method is employed, and the equation of motion is determined using Lagrange's equations.

2. Composite shaft

The composite shaft is obtained by winding several layers of embedded fibers on a mandrel. Each layer has an orthotropic mechanical behaviour (see Fig. 1).

The generalized Hooke law for an orthotropic material is given by:

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{1}$$

where $\{\sigma\}$ and $\{\varepsilon\}$ are, respectively, the stress and the strain fields, and $[Q]$ is the material stiffness matrix. When linked to the orthotropic axis, each layer can be characterized by a plane stress state. So, one gets:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 & 0 & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 & 0 & 0 \\ 0 & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} \tag{2}$$

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