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Static and dynamic behaviour of nonlocal elastic bar using integral strain-based and peridynamic models

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ABSTRACT

The static and dynamic behaviour of a nonlocal bar of finite length is studied in this paper. The nonlocal integral models considered in this paper are strain-based and relative displacement-based nonlocal models; the latter one is also labelled as a peridynamic model. For infinite media, and for sufficiently smooth displacement fields, both integral nonlocal models can be equivalent, assuming some kernel correspondence rules. For infinite media (or finite media with extended reflection rules), it is also shown that Eringen's differential model can be reformulated into a consistent strain-based integral nonlocal model with exponential kernel, or into a relative displacement-based integral nonlocal model with a modified exponential kernel. A finite bar in uniform tension is considered as a paradigmatic static case. The strain-based nonlocal behaviour of this bar in tension is analyzed for different kernels available in the literature. It is shown that the kernel has to fulfil some normalization and end compatibility conditions in order to preserve the uniform strain field associated with this homogeneous stress state. Such a kernel can be built by combining a local and a nonlocal strain measure with compatible boundary conditions, or by extending the domain outside its finite size while preserving some kinematic compatibility conditions. The same results are shown for the nonlocal peridynamic bar where a homogeneous strain field is also analytically obtained in the elastic bar for consistent compatible kinematic boundary conditions at the vicinity of the end conditions. The results are extended to the vibration of a fixed–fixed finite bar where the natural frequencies are calculated for both the strain-based and the peridynamic models.

1. Introduction

Nonlocal models are continuum models that are able to account for the change of scale in the analysis of a structure that contains some microstructure patterns. Among integral-based nonlocal models, strain-based and relative displacement-based models have been both developed in the literature. Strain-based nonlocal models relate the stress to the strain through an integral operator valid in the whole range of the solid, whereas relative-based displacement models expressed the balance equation through an integral operator of the displacement difference, which avoids the calculation of the strain through a gradient operator. Strain-based nonlocal models have emerged in the 1960's for bridging lattice mechanics with engineering

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continuum models, especially (but not only) to capture the wave dispersive properties of crystal materials (see for instance [1,2] or [3]). These models have been further cast in a consistent thermodynamics framework (see for instance [4] or more recently [5]). Altan [6] studied the uniqueness of the boundary value problem of strain-based nonlocal elasticity static problems. Strain-based integral nonlocal elasticity theories are widely reviewed in the seminal book of Eringen [7] – see the historical analysis of Maugin [8] on the topic. Relative displacement-based models have been introduced by Silling [9] through the terminology of peridynamic models, and then widely developed by the same author and his co-authors for several engineering applications ([10,11] for instance). These models can also be understood as the continualization of lattice models that account to short and long-range interactions, thus being also associated with the concept of physically-based nonlocal models [12]. A fractional peridynamic model has been recently reported by Lazopoulos [13].

Some mathematical properties have to be fulfilled by the kernel for both integral approaches, including the strain-based or relative displacement-based approaches. The kernel has to fulfil some normalization and end compatibility conditions in order to preserve the uniform strain field associated with this homogeneous stress state. For infinite media, analytical and numerical solutions have been found for various kernels, and the discussion of compatible boundary conditions is avoided. For finite-body problems, the kernel associated with the integral model has to be compatible with the boundary conditions of the problem. This incompatibility between the natural boundary conditions and the induced kernel-dependent boundary conditions prevent the use of some kernels including some elementary exponential-based kernels (as detailed by Fernández-Sáez et al. [14] or by Romano et al. [15] for nonlocal integral beam models). Many nonlocal strain measures developed in the literature violate the nonlocal invariance of the uniform strain field, which can be physically questionable. Consequently, there is a need to select appropriate kernels for engineering applications and to find relevant solutions in some benchmark cases for simple nonlocal structural mechanics applications.

The same remarks hold for relative displacement-based nonlocal models, whose available solutions have been mostly derived in statics and in dynamics for infinite one-dimensional media by Silling et al. [10], Mikata [16] or Bažant et al. [17]. An exception is the recent paper of Nishawala and Ostoja-Starzewski [18], who obtained an analytical solution for a finite bar in tension under various distributed loadings. Nishawala and Ostoja-Starzewski [18] also discussed some correction effects at the vicinity of the finite bar, by introducing some distributed load that affects the homogeneous stress state configuration of the problem. In a certain sense, the difficulties pointed out in the strain-based integral model are not avoided in the relative displacement-based models, and some clarifications are needed for both models.

In this paper, we explore some possible link between nonlocal elasticity, peridynamics theory also labelled as a nonlocal relative displacement-based theory and lattice mechanics. The statics and the vibration of a finite bar is investigated, and some exact analytical solutions are derived for possible benchmark testing.

2. General equations of integral-based nonlocal models for one-dimensional problems

A one-dimensional strain-based nonlocal model can be introduced from the following integral operator (see for instance [7]):

$$N(x) = EA \int_0^L G(x, y)\varepsilon(y)\,\mathrm{d}y \tag{1}$$

for a finite bar of length L, where N is the normal force, ε is the axial strain, E is the Young modulus, A is the area and $G(x, y)$ is the nonlocal kernel of the strain-based nonlocal model, which should verify the translational invariance principle for homogeneous isotropic media [7]:

$$G(x, y) = g(\xi) = g(-\xi) \quad \text{with } \xi = y - x \tag{2}$$

G has the dimension of the inverse of a length, i.e. L^{-1}. The normalization procedure, in order to leave the uniform strain unchanged, for finite structural elements (see also [7] or [19]; or [40] for the discussion of the criterion for finite solids) can be written as:

$$\int_0^L G(x, y)\,\mathrm{d}y = 1 \tag{3}$$

To avoid any difficulties at the limit of the finite domain, and following the methodology that is applied to lattice problems which may be also investigated within nonlocal mechanics (see also [7]), it is sometimes preferred to extend the finite domain outside its domain of definition following some symmetrical properties or periodic properties, leading to the nonlocal normal force strain-based definition:

$$N(x) = EA \int_{-\infty}^{+\infty} G(x, y)\varepsilon(y)\,\mathrm{d}y \tag{4}$$