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Classical and sequential limit analysis revisited

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ABSTRACT

Classical limit analysis applies to ideal plastic materials, and within a linearized geometrical framework implying small displacements and strains. *Sequential* limit analysis was proposed as a heuristic extension to materials exhibiting strain hardening, and within a fully general geometrical framework involving large displacements and strains. The purpose of this paper is to study and clearly state the precise conditions permitting such an extension. This is done by comparing the evolution equations of the full elastic–plastic problem, the equations of classical limit analysis, and those of sequential limit analysis. The main conclusion is that, whereas classical limit analysis applies to materials exhibiting elasticity – in the absence of hardening and within a linearized geometrical framework –, sequential limit analysis, to be applicable, strictly prohibits the presence of elasticity – although it tolerates strain hardening and large displacements and strains. For a given mechanical situation, the relevance of sequential limit analysis therefore essentially depends upon the importance of the elastic–plastic coupling in the specific case considered.

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1. Introduction

The classical theory of limit analysis needs little introduction. This very useful theory is however based on the restrictive assumptions of ideal plasticity and linearized geometrical framework – small displacements and strains.

An essentially heuristic proposal was made by Yang [1] to alleviate these unhappy restrictions. The proposed extended framework, named *sequential limit analysis*, incorporates the effects of both strain hardening and geometric changes, and has been developed in close conjunction with numerical methods. Its goal is to address the following three questions, the answers to which fully define the evolution of the structure in time:

(1) For a given distribution of hardening parameters and a given geometrical configuration, *when does global plastic flow of the structure occur* – that is, what are the necessary conditions on the load governing this flow?
(2) Supposing such a flow takes place, *how does it occur* – that is, what is the mode of deformation of the structure?
(3) Again assuming a deforming structure, *how do the distribution of hardening parameters and the geometrical configuration change in time*?

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In order to answer questions (1) and (2), the principle of sequential limit analysis consists in considering a hardenable material in large strain as a discrete sequence of different, successive ideal plastic materials occupying different, successive geometrical configurations. In this approach, at a given discretized instant, the hardening and the geometry are considered as momentarily fixed. The hardenable material, considered within a fully general geometrical framework, then behaves like an ideal plastic material, considered within a linearized geometrical framework, the role of the previous hardening being merely to modify the instantaneous local yield criterion and the plastic flow rule, and that of the previous geometry changes to fix the present geometrical configuration. An "overall yield locus", supplemented by an "overall associated plastic flow rule", can then be determined using the standard theorems of classical limit analysis.

To answer question (3), the evolutions of the distribution of hardening parameters and the geometrical configuration are deduced *a posteriori*, by approximately updating these quantities using the trial velocity field used in the limit analysis, integrated within a time step.

The proposal has been received with some enthusiasm and sequential limit analysis has been used for various applications; see for instance the works [2–6], to quote just a few.

However, the mechanical foundations of sequential limit analysis have remained vague and to some large extent unclear, and this may prompt doubts about the overall validity of the method. How can one ignore, even momentarily, the evolutions of the hardening parameters and the geometry, whereas in reality they change constantly? Clearly, in order to settle such a question, it is necessary to scrutinize the connection between the evolution equations of the full elastic–plastic problem and the simpler equations of sequential limit analysis.

It will also be necessary to investigate the connection with the equations of classical limit analysis, because sequential limit analysis uses the results and methods of classical limit analysis, which is justified only provided the equations of both theories are analogous. Re-examining the equations of classical limit analysis will also be useful to re-assess, and remind the reader of the role played by elasticity in this theory. Indeed although Drucker et al. [7] have clearly shown that classical limit analysis perfectly tolerates the presence of elasticity, the memory of their work seems to have somewhat faded in time.

The paper is organized as follows.

- Section 2 is devoted to a short summary of the classical works the classical works of Drucker et al. [7] and Hill [8]. The discussion concentrates on those results of the theory actually used in its practical application. The discussion of the role of elasticity follows the work of Drucker et al. [7].
- Section 3 then discusses the foundations of sequential limit analysis, exploiting an analogy of its equations with those of classical limit analysis and using some, if not all, elements of Section 2. Special attention is again paid to the role of elasticity. The precise conditions of applicability of sequential limit analysis are made clear.
- Finally Section 4 considers, as a typical example, the case of porous ductile materials, which involves all possible complexities: elasticity, plasticity, strain hardening, and large strains. Using the results of Section 3, we discuss the applicability of sequential limit analysis to the derivation, through homogenization, of micromechanically-based models.

2. Classical limit analysis

2.1. Hypotheses and notations – equations of the problem

We consider a body Ω made of some *elastic–ideal plastic* material obeying the plastic flow rule associated with the yield criterion through Hill's normality property. The elastic–plastic evolution problem is considered within a *linearized geometrical framework*, so that its equations are written using the initial position vector $\mathbf{X}$, the linearized strain tensor $\boldsymbol{\epsilon}$, and the Cauchy stress tensor $\boldsymbol{\sigma}$. With these hypotheses these equations read as follows (disregarding body forces for simplicity):

$$\begin{cases} \mathbf{div}_{\mathbf{X}}\boldsymbol{\sigma} = \mathbf{0} & \text{equilibrium} \\ \dot{\boldsymbol{\epsilon}} \equiv \frac{1}{2}\left[\mathbf{grad}_{\mathbf{X}}\dot{\mathbf{u}} + (\mathbf{grad}_{\mathbf{X}}\dot{\mathbf{u}})^{\mathsf{T}}\right] & \text{definition of } \dot{\boldsymbol{\epsilon}} \\ \dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^{\mathrm{e}} + \dot{\boldsymbol{\epsilon}}^{\mathrm{p}} & \text{additive decomposition of } \dot{\boldsymbol{\epsilon}} \\ \dot{\boldsymbol{\epsilon}}^{\mathrm{e}} = \mathbf{S} : \dot{\boldsymbol{\sigma}} & \text{elasticity law} \\ \dot{\boldsymbol{\epsilon}}^{\mathrm{p}} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}) & \text{plastic flow rule obeying the normality property} \\ f(\boldsymbol{\sigma}) \le 0,\ \dot{\lambda} \ge 0,\ f(\boldsymbol{\sigma})\dot{\lambda} = 0 & \text{Kuhn–Tucker's complementarity conditions} \\ B.C. & \text{boundary conditions} \end{cases} \tag{1}$$

In these equations, $\dot{\mathbf{u}}$ denotes the velocity vector ($\mathbf{u}$ is the displacement vector), $\dot{\boldsymbol{\epsilon}}^{\mathrm{e}}$ the elastic strain rate tensor, $\dot{\boldsymbol{\epsilon}}^{\mathrm{p}}$ the plastic strain rate tensor, $\mathbf{S}$ the elastic compliance tensor, $f(\boldsymbol{\sigma})$ the yield function (depending only on $\boldsymbol{\sigma}$ in the absence of hardening), and $\dot{\lambda}$ the plastic multiplier. The boundary conditions, symbolized by the letters "*B.C.*", need not be stated explicitly here; it will suffice to say that they depend linearly upon a finite number of load parameters $Q_1, Q_2, \ldots, Q_N$, collectively defining a "load vector" $\mathbf{Q}$. The conjugate kinematic parameters $q_1, q_2, \ldots, q_N$, collectively defining a "kinematic vector" $\mathbf{q}$, are defined in such a way that the virtual power of the external forces is $\sum_{i=1}^{N} Q_i\dot{q}_i = \mathbf{Q}\cdot\dot{\mathbf{q}}$.

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