C. R. Mecanique ••• (••••) •••–•••

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The legacy of Jean-Jacques Moreau in mechanics

The Contact Dynamics method: A nonsmooth story

La méthode de la dynamique des contacts, histoire d'une mécanique non régulière

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ARTICLE INFO

Article history:
Received 20 May 2017
Accepted after revision 11 October 2017
Available online xxxx

Keywords:
Nonsmooth dynamics
Shock
Coulomb law
Contact Dynamics
Discrete element method

Mots-clés :
Dynamique non régulière
Chocs
Loi de Coulomb
Dynamique des contacts
Méthode par éléments discrets

ABSTRACT

When velocity jumps are occurring, the dynamics is said to be nonsmooth. For instance, in collections of contacting rigid bodies, jumps are caused by shocks and dry friction. Without compliance at the interface, contact laws are not only non-differentiable in the usual sense but also multi-valued. Modeling contacting bodies is of interest in order to understand the behavior of numerous mechanical systems such as flexible multi-body systems, granular materials or masonry. These granular materials behave puzzlingly either like a solid or a fluid and a description in the frame of classical continuous mechanics would be welcome though far to be satisfactory nowadays. Jean-Jacques Moreau greatly contributed to convex analysis, functions of bounded variations, differential measure theory, sweeping process theory, definitive mathematical tools to deal with *nonsmooth* dynamics. He converted all these underlying theoretical ideas into an original nonsmooth implicit numerical method called Contact Dynamics (CD); a robust and efficient method to simulate large collections of bodies with frictional contacts and impacts. The CD method offers a very interesting complementary alternative to the family of smoothed explicit numerical methods, often called Distinct Elements Method (DEM). In this paper developments and improvements of the CD method are presented together with a critical comparative review of advantages and drawbacks of both approaches.

RÉSUMÉ

Lorsque des sauts de vitesse se produisent, la dynamique est dite non régulière. Par exemple, dans les collections de solides supposés rigides rentrant en contact, les sauts sont causés par les chocs et le frottement sec. L'absence de déformabilité fait que les lois de contact sont, non seulement non différentiables au sens usuel, mais aussi multi-valuées. Élaborer des modèles de solides en contact est un moyen de comprendre le comportement de nombreux systèmes mécaniques tels que les systèmes multi-corps flexibles, les matériaux granulaires ou les maçonneries. Les matériaux granulaires se comportent de manière étrange, soit comme des solides, soit comme des fluides, et une

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https://doi.org/10.1016/j.crme.2017.12.009

description dans le cadre de la mécanique classique des milieux continus, qui serait souhaitable, est loin d'être encore satisfaisante. Jean-Jacques Moreau a contribué, de façon fondamentale, à l'analyse convexe, à la théorie des fonctions à variations bornées et des mesures différentielles ainsi qu'au processus de rafle, outils mathématiques décisifs pour traiter *la dynamique non régulière*. Il a converti ces idées théoriques sous-jacentes en une méthode numérique originale appelée *Contact Dynamics* (CD), qui est une méthode non régulière implicite et aussi une méthode robuste et efficace pour simuler de larges collections de solides avec du contact frottant et des impacts. Le méthode CD offre une alternative très intéressante à la famille de méthodes usuelles régularisées explicites, comme la méthode des éléments distincts (DEM). Dans cet article, des développements et des perfectionnements de la méthode CD sont présentés ainsi qu'une étude critique comparative des avantages et inconvénients des deux approches.

1. Introduction

The generic label (DEM) Discrete Elements Methods, also called Distinct Elements Methods, refers to methods that, oppositely to Finite Elements Methods (FEM) dedicated to the description of media in the frame of continuous mechanics, consider a sample as an assembly of distinct bodies. Nowadays, such methods are widely used in the numerical modeling of divided materials and structures. Natural applications concern the simulation of granular materials, suspensions, fractured materials, masonries, rock mass, etc. in domains such as geophysics, mining, chemical engineering, civil engineering, biomechanics, etc. DEM are also used for their capability to represent the various states of a collection of solids (gas, fluid, solid) and to represent some phase changes (solid to solids and vice versa) in the spirit of meshless methods [1] or particle methods [2,3]. But such methods are also applied in multi-body systems such as mechanisms and robotics. Usually, in such methods, one considers collections of rigid bodies, subject to interaction laws such as frictional contact laws that are steep laws.

A number of leading methods are derived from the pioneering work of Cundall [4] actually referred to the generic label DEM. This work may be considered also as a modification of the genuine Molecular Dynamics method as proposed by Allen and Tidsley [5]. Since such methods are particularly pragmatic, steep frictional contact laws are modeled as nonlinear laws using some regularization techniques, and explicit time integrators are used to cope with the nonlinear behavior. At the end, such methodology leads to a set of uncoupled linear equations that can be solved straightforwardly. The name DEM is commonly referring to those *smoothed and explicit* methods. The weaknesses of such methods come from their principle: small time steps are mandatory due to explicit time integrators, the choice of relevant parameters may be tricky, since they are used to manage several phenomena: contact condition, macroscopic mechanical response, dynamical properties, etc. In particular, in order to ensure the stability of explicit schemes, it is necessary to introduce some damping, either generated by the frictional contact laws, either as a numerical trick. There exist also several “confidential” methods such as Discontinuous Deformation Analysis (DDA) [6,7], or Discrete Fracture Network (DFN) [8]. For a small number of objects, the classical Finite Element Method (FEM) can also manage contact problems [9].

Jean-Jacques Moreau introduced the (CD) Contact Dynamics method during the year 1984. It is inspired by a formulation of unilateral contact, shock laws, Coulomb friction, through Convex Analysis. The contacting laws are thus non differentiable steep laws. They are managed with an implicit method using a Non-Linear Gauss–Seidel algorithm (NLGS) at each step. These laws account roughly for the main features of contact and friction and are relevant in multi-bodies collections where sophisticated laws cannot be exhibited for sure. The method uses large time steps, but each time step is time consuming. So oppositely to the above smoothed and explicit DEM method, the CD method is a *nonsmooth and implicit* method. Note that implicit methods enable to compute correctly equilibrium states, which is not always the case with explicit methods. The method can also conserve, with a suitable choice of parameters, the total energy of the system in discrete time. In this paper, we shall discuss the pros and cons of the CD method with respect to classical smoothed DEM.

2. Moreau contribution to Contact Dynamics genesis

First, it is to be remembered that Jean-Jacques Moreau was earlier concerned with fluid mechanics. He left an original result concerning the helicity invariant in perfect fluid dynamics (1961) [10].

Jean-Jacques Moreau was introducing himself as involved in mechanics, claiming that he was using mathematics just enough for his mechanical purpose. Actually he was the author of highly sophisticated concepts in mathematics mainly in the field of measure theory. He developed definitely the theory of locally bounded variation functions [11], the proper mathematical setting for the nonsmooth dynamics and the sweeping process theory [12–15]. After the works of the pioneers, H. Minkowski and M.W. Fenchel, he set up Convex Analysis, a theory also developed at the same time by R.T. Rockafellar; see for instance the numerous references in the famous book [16] to the results of Jean-Jacques Moreau. Convex Analysis is the proper tool to deal with nonsmooth mechanics i.e. mechanics where the behavior laws are not differentiable in the

Please cite this article in press as: F. Dubois et al., The Contact Dynamics method: A nonsmooth story, C. R. Mecanique (2017), https://doi.org/10.1016/j.crme.2017.12.009