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Limit analysis, rammed earth material and Casagrande test

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ABSTRACT

The present paper is concerned with the simulation of the Casagrande test carried out on a rammed earth material for wall-type structures in the framework of Limit Analysis (LA). In a preliminary study, the material is considered as a homogeneous Coulomb material, and existing LA static and kinematic codes are used for the simulation of the test. In each loading case, static and kinematic bounds coincide; the corresponding exact solution is a two-rigid-block mechanism together with a quasi-constant stress vector and a velocity jump also constant along the interface, for the three loading cases. In a second study, to take into account the influence of compressive loadings related to the porosity of the material, an elliptic criterion (denoted Cohesive Cam–Clay, CCC) is defined based on recent homogenization results about the hollow sphere model for porous Coulomb materials. Finally, original finite element formulations of the static and mixed kinematic methods for the CCC material are developed and applied to the Casagrande test. The results are the same than above, except that this time the velocity jump depends on the compressive loading, which is more realistic but not satisfying fully the experimental observations. Therefore, the possible extensions of this work towards non-standard direct methods are analyzed in the conclusion section.

1. Introduction

Earth materials have been attracting numerous scientific researches because of the low embodied energy and of a positive hygrothermal behavior. The present paper deals with the simulation of the Casagrande test carried out in our laboratory on a rammed earth material denoted “pisé” in French. The novelty of this study concerns the use of both Limit Analysis (LA) methods, firstly when considering the material as obeying the Coulomb criterion, and secondly when using a homogenization model for porous materials to define an elliptical criterion (of Cam–Clay type) in an attempt to better represent the earth material for finite structures such walls; in each case, the behavior of the material in the experimental tests is analyzed by *ad hoc* finite element modelings. All resulting codes lead to conic optimization problems solved with the MOSEK commercial code. It is worth noting here that numerical and analytical solutions about direct computation of limit loads (i.e. in the LA framework) with Cam–Clay-type criteria do not exist in the literature, at least up to our knowledge.

The paper is organized as follows. First, the results of a preliminary study, where the rammed earth is considered as a homogeneous Coulomb material and by using existing LA codes, are briefly presented. For all loading cases, the final static

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and kinematic approaches give quasi-identical results in terms of loading parameters, with a two-rigid block kinematic solution with a constant velocity jump, and a quasi-constant stress vector along the shearing plane. However, the observed dilatancy is not depending on the intensity of the vertical loading for a given velocity on the lower half-box: such a feature does not seem realistic when considering homogenization results for materials with a non-negligible porosity.

In a second part, to take into account this porous character, an elliptic criterion, here denoted Cohesive Cam–Clay criterion (CCC), is defined on the basis of the homogenization approach called the hollow sphere model of references [1], [2], and [3]. In the last section, original static and kinematic LA codes are developed for this CCC criterion and used for the shear test problem, the corresponding results being discussed in the final part of the paper.

2. The Limit Analysis methods, briefly

The Limit Analysis theory aims to determine the limit loads of a material system that obeys a criterion of convex plasticity and the associated flow rule. This theory allows obtaining a framework for these limit loads via two separate approaches and a mixed approach. For the two separate approaches, the first method, or static method, provides a lower bound of the limit load. The second method, called kinematic (classical), gives an upper bound. The first one contains only the stresses as variables, and the second one only the velocities. The mixed approach involves both types of variables and requires only the expression of the plasticity criterion as a characteristic of the material. In addition, under certain conditions verified here, it also leads to rigorous upper bounds.

After discretization of the mechanical system in finite elements, the resulting problems lead to optimize a linear functional under linear and convex constraints. Since the early 2000s, the conic optimization codes (such as MOSEK, [4]) made it possible to solve directly these non-linear optimization problems with a very good efficiency. In the present paper, the MOSEK conic code is also used for solving the problems arising from an improved version of the criterion proposed in [2], first for identifying the criterion itself, then for the FEM simulations of the experimental tests.

3. The Casagrande test and its preliminary modelization as a Coulomb material: results

This section aims to present the mechanical and numerical environment available to simulate the shear test with the Casagrande box (Fig. 1), for the Coulomb material. The presentation of the experimental apparatus, too long to be detailed here, can be found in references [5,6].

3.1. Mechanical modeling of the Casagrande test

Schematically, the test (cf. Fig. 1, left) consists in applying a vertical force N by means of a metallic plate, and the shearing at the interface of the two metal half-boxes (of square section) is carried out via the horizontal displacement of the lower half-box. The upper half-box is horizontally blocked. The lower half-box is laid on rollers in order to make the sliding resistance negligible. In the figure is plotted the median section of the apparatus, where metal parts are in black and Teflon in green. The horizontal velocity V, here positive, of the lower half-box (A′B′C′D′) is noted V_F in the following, and the shear plane is noted GH.

Taking into account that the vertical parts and the bottom of the box are provided with a thin Teflon layer, it can be assumed that the corresponding interfaces (EG, FH and GIJH in the figure) are smooth. The metal parts of the box are undeformable compared to the material, which is assumed homogeneous and isotropic. The displacement of the box on the z axis is zero, and the axial load N is applied in the middle of the upper plate. Due to the symmetries and the boundary conditions of the problem, a plane strain approach is used, where the (x, y) plane is that of the figure, with the velocity field in the material $(u_x(x, y), u_y(x, y), u_z = 0)$. For the present problem, the extension of the stress field solution to the 3D case is easy (with $\tau_{yz} = \tau_{zx} = 0$, and $\sigma_z(x, y)$ only constrained by the plasticity criterion). Similarly, the displacement velocity field is directly extensible in 3D with $u_z(x, y) = 0$: the exact solution in 2D will also be exact from a three-dimensional point of view, and thus for the 3D test box.

As a consequence, Fig. 1 represents the plane strain test simulated in the context of the limit analysis. Let us assume *a priori* the Coulomb law of friction on the GH interface. Since the velocity jump in the tests is non-zero along GH, the relation $|\tau| = c - \sigma_n \tan\phi$ is verified (with positive traction stress, and c, ϕ constants); the equilibrium conditions of the two half-boxes (Fig. 1) classically give:

$$F = \int_{GH} \tau_{nt}\, \mathrm{d}S = cL - \tan\phi \int_{GH} \sigma_y\, \mathrm{d}S = cL + N\tan\phi \quad (1)$$

where L is the length of the segment GH, and F the force associated with the imposed velocity V_F. The Casagrande test returns directly the cohesion and the friction angle of the *a priori* assumed Coulomb criterion without any hypothesis about the plastic flow rule of the material. In terms of the LA static approach, since all the corresponding stress fields (including the one of the exact LA solution) must be statically admissible, these fields obey the local equilibrium conditions, and consequently the equilibrium of the parts, used above, is also satisfied; Relation (1) gives the exact LA solution *in terms of the loading parameters N and F*.

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