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Effects of modal truncation and condensation methods on the Frequency Response Function of a stage reducer connected by rigid coupling to a planetary gear system

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A B S T R A C T

The present paper is aimed at the application of a substructure methodology, based on the Frequency Response Function (FRF) simulation technique, to analyze the vibration of a stage reducer connected by a rigid coupling to a planetary gear system. The computation of the vibration response was achieved using the FRF-based substructuring method. First of all, the two subsystems were analyzed separately and their FRF were obtained. Then the coupled model was analyzed indirectly using the substructuring technique. A comparison between the full system response and the coupled model response using the FRF substructuring was investigated to validate the coupling method. Furthermore, a parametric study of the effect of the shaft coupling stiffness on the FRF was discussed and the effects of modal truncation and condensation methods on the FRF of subsystems were analyzed.

1. Introduction

Transmission systems are widely used in manufacturing applications due to their power advantages, decreased cost, and high efficiency. The multi-stage gears generally have larger reduction ratios and greater load transmissions. Dynamic models of transmission systems, including parallel and planetary gears, were widely investigated in literature [1–3]. Several issues have made the parallel gear specially interesting. The effects of bearing flexibilities and axial vibrations of a parallel gear were studied in [4,5]. Planetary gears are used in transmission applications such as wind turbines and helicopters, requiring a higher transmission torque. A great deal of research focused on the static and dynamic models of a planetary transmission system, created by adopting the lumped mass method [6] and the finite element (FE) model [7]. The dynamic behavior of a planetary transmission system was investigated in [8,9]. The sensitivity of natural frequencies and vibration modes in compound planetary gears were analyzed in [10]. On the other hand, a lot of research investigated the dynamic behavior of a gearbox model formed by two transmission systems which are the planetary and the parallel gears [11–14]. Such

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transmission systems are widely used in wind turbine gearboxes (WTG). Peeters et al. [15] developed the flexible multibody dynamics model of a wind turbine transmission system in multibody dynamics software and studied its natural frequencies and vibration modes. Feng and Zuo [16] investigated the vibration signal models for planetary gearbox fault diagnosis.

The dynamic analysis of complex transmission systems requires much more Central Processing Unit (CPU) and computation time. In order to simplify this analysis and save their computational time, some researchers were interested in the study of the substructuring method and component mode synthesis [17,18]. These methods are used to build the dynamic response of complex systems by assembling the dynamic models from its subsystems. These subsystems can be expressed by spatial mass, stiffness, damping data, modal data, or receptances (dynamic compliance). The major concept of the substructuring method is to use the Frequency Response Function (FRF) for computing the vibration response of the free subsystem. The overall coupled system response is then composed by the dynamic compliance theory [19] thanks to the dynamic stiffness terms at the coupling coordinates of the subsystems [20]. This kind of coupling can be either rigid or flexible with the dynamic stiffness [21]. Some techniques of substructuring, as coupling and decoupling methods were proposed in [22–24]. Nevertheless, the coupling method applied to gears systems is quite limited in the literature. Unlike coupling, the decoupling method was developed for the transmission system in [23,25]. In the literature, the coupling method was investigated for coupling two subsystems in other applications such as automotive ones in [21,26] and rotating systems in [22] based on the FRF. In this context, the analytic receptance coupling method was developed by Bishop and Johnson [18]. The greatest advantage of this approach is its ability to increase efficiency and decrease the computation time. Furthermore, the use of FRF provides a common basis to combine subsystems of numerical and experimental origins.

The condensation method and the modal truncation are used in the analysis of complex systems to reduce the computation time and the problem size. Different reduction modes approaches were proposed in the literature to construct the reduction basis of a subsystem such as the condensation and the truncated modes. The dynamic condensation was applied to investigate the influences of neighboring subsystems and coupling on the FRF of each subsystem in [25,27]. The modal truncation problem faced with in the responses of some applications was treated in [26,28–31], and the effect of truncation modes was analyzed in [28]. A modal based approach for decoupling associated with modal truncation was presented in [30]. In addition, Ambrogio and Fregolent [30] studied the effect of modal truncation on the natural frequencies by decreasing the number of modes. Suarez [32] introduced a force derivative method to analyze the effect of the truncated higher modes in the representation of the response of the substructures. Only the modes located in the interesting frequency range were considered in the dynamic analysis. In fact, modal truncation was used to express the contribution of the out-of-range modes on the FRFs of each subsystem.

In this paper, the application of the substructuring technique was investigated for the analysis of the vibration signals of a transmission system, consisting of two gear stages, which are the parallel and planetary gear systems. The present method was developed in order to reduce the size of the computational problem and the complexity of the dynamic model. In fact, the analysis of the global system is more difficult than that of the local dynamic behavior. If one subsystem is replaced by another with a known FRF, this method becomes useful and allows a rapid computation of the FRF of the whole system. In addition, the substructuring method is interested in some applications of damage detection in gear systems [16]. The modal truncation was investigated to obtain a consistent basis for truncation modes of substructures in the vibratory analysis of a complex system.

The remaining of this paper is organized as follows. The dynamic models of planetary gear and parallel stage are described in section 2. The theoretical principles of the substructuring method are treated in section 3. The results of the FRF coupling method were compared to those of the FRF of the full system in section 4. The effects of condensation and modal truncation were investigated in section 5 before drawing the major conclusions in the final section.

2. Motion equation

The studied transmission system consists of two subsystems: a first subsystem A connected to a second subsystem B by a rigid coupling, corresponding respectively to a parallel and to a planetary gear stage. The models of the parallel stage and the spur planetary gears were established by adopting the lumped parameter model (Fig. 1). The parallel stage gear is similar to the model proposed by Kahraman [5]. Nevertheless, eight DoFs were used to describe the gear system behavior rather the ten DoFs adopted by [5]. The displacements of the radial gears were not taken into account in the reference model. Both gears are seated on two rigid shafts, supported by flexible bearings.

The gears are modeled by rigid disks of masses m_1 and m_2, polar mass moments of inertia about z-axis J_1 and J_2 and diametral mass moments of inertia about the y-axis I_1 and I_2. The second gear is connected to the planetary gear. The gear-mesh is represented by linear springs along the action line. The gear mesh stiffness is assumed constant. The radial and the axial stiffness bearings are denoted by k_{yj} and k_{zj} respectively. The second shaft of the reducer is connected to the sun shaft by a rigid coupling of inertia I_A. The shafts are supported by bearings, modeled by linear springs. The gears are modeled by concentrated masses. The planetary gear train components such as ring, carrier, sun, and N planets are assumed to be rigid bodies. Each one of these components of the planetary train has three degrees of freedom: two translations and one rotation. The rotation coordinates are denoted by $w_j = r_j\theta_j$ $(j = c, s, r)$, where θ_j is the rotation angle, r_j is the base circle radius of the sun. In Fig. 2, u_i, v_i, and w_i $(i = c, s, r)$ are respectively the displacements of the sun gear, the carrier and the ring in two radials and rotational directions. The planetary gear system was investigated in [33]. k_{cu} and k_{su} are the support stiffness of the carrier and the sun gears in the u direction, respectively. k_{cv} and k_{sv} are the support stiffness