C. R. Mecanique ••• (••••) •••–•••

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Stochastic model reduction for robust dynamical characterization of structures with random parameters

Martin Ghienne, Claude Blanzé, Luc Laurent *

Laboratoire de mécanique des structures et des systèmes couplés, Conservatoire national des arts et métiers, case courrier 2D6R10, 2, rue Conté, 75003 Paris, France [1]

ARTICLE INFO

Article history:
Received 25 July 2017
Accepted 18 September 2017
Available online xxxx

Keywords:
Random eigenvalue problems
Statistical distributions
Linear stochastic systems
Perturbation
Simplified resolution method
Proximity factor

ABSTRACT

In this paper, we characterize random eigenspaces with a non-intrusive method based on the decoupling of random eigenvalues from their corresponding random eigenvectors. This method allows us to estimate the first statistical moments of the random eigenvalues of the system with a reduced number of deterministic finite element computations. The originality of this work is to adapt the method used to estimate each random eigenvalue depending on a global accuracy requirement. This allows us to ensure a minimal computational cost. The stochastic model of the structure is thus reduced by exploiting specific properties of random eigenvectors associated with the random eigenfrequencies being sought. An indicator with no additional computation cost is proposed to identify when the method needs to be enhanced. Finally, a simple three-beam frame and an industrial structure illustrate the proposed approach.

1. Introduction

Requirements on system performance are increasingly stringent, which leads us to question the design rules currently used in structural engineering. The use of safety factors that are often inconsistent and confusing is no longer sufficient in many leading-edge fields, and it is necessary to decrease the gap between the observed behavior of a structure and predictions from numerical simulations based on deterministic models. In order to accurately predict the real behavior of a structure, the variability of the observed behavior needs to be modeled. This variability is mainly due to the system parameter's randomness. The probabilistic approach is adapted to numerical resolution [1,2]. This is thus the framework adopted for this work.

The aim is to robustly characterize the vibrational response of a structure in a random manner from a finite element model of the structure. We are particularly looking for the eigenspace characterization of linear systems with dynamic properties considered as random variables. Methods based on statistical sampling provide a good framework to solve the random dynamic problem; nevertheless, they need intensive computation to remain accurate [3,4]. The performance of this kind of method strongly depends on the quality of the random number generator and the total computational cost increases dramatically with the cost of one deterministic case. For these reasons, reduced-order models in a context such as PGD [5], for instance, or non-sampling methods have been introduced. Along the latter, different techniques have been developed over

* Corresponding author.
E-mail addresses: martin.ghienne@lecnam.net (M. Ghienne), claude.blanze@lecnam.net (C. Blanzé), luc.laurent@lecnam.net (L. Laurent).
[1] www.lmssc.cnam.fr

https://doi.org/10.1016/j.crme.2017.09.006
1631-0721/

the last few decades. Two particular approaches are mainly used in the literature to approximate the statistical properties of the response of a random system: the perturbation and the spectral methods.

The perturbation method is based on an approximation of the random variable of interest through the truncation of its Taylor expansion. Its implementation is pretty easy, but as high-order perturbation terms are computationally intensive, the expansion is generally limited to the second order. Moreover, variations of the system parameters should remain small to guarantee the accurate estimation of statistical moments [6]. Generally, random variables are expanded by their Taylor series about their mean value. For example, Collins and Thomson [7] estimate statistics of random eigenvalues and eigenvectors. Adhikari and Friswell [8] propose to expand around an optimal point in order to better approximate the first moments of random eigenvalues. Nair and Keane [9] use a perturbation method to define an approximation subspace and to estimate the system's random eigenvectors.

The Spectral Stochastic Finite Element Method (SSFEM) was introduced by Ghanem and Spanos in [10], inspired by Wiener's works [11]. The method is based on a discretization of the random variables of interest among a finite random space. The random variables are decomposed on the basis of orthogonal polynomials in terms of the multi-dimensional random variable with a specific probability distribution. This basis is called the polynomial chaos. For problems with Gaussian random input parameters, the best-suited basis consists of a set of multidimensional Hermite polynomials [12]; it ensures fast convergence and accurate approximation of the random variables. If the input parameters are not Gaussian, other proper bases have been developed to ensure an optimal convergence [13]. Once the decomposition basis is chosen, the coefficients of the decomposed random variable need to be computed. For this purpose, a Galerkin-based method described in [14] allows us to estimate the coefficients using Monte Carlo sampling. This method suffers from its sensibility to the quality of the random number generator [15,16] and becomes quickly computationally intensive in the case of high-order polynomials or of a high number of random parameters. To overcome these drawbacks, Ghanem and Ghosh [14] propose another Galerkin-based method for reducing the problem to a set of deterministic non-linear equations.

In order to characterize the eigenspace of a system with random dynamic properties, polynomial chaos methods give a general and accurate framework, but their implementation is rather complex. On the other hand, perturbation methods are easy to implement, but their intrinsic assumptions limit their application to academic problems or small variations of the input parameters. Based on these techniques, Pascual and Adhikari [17] have proposed and compared methods hybridizing perturbation approach and polynomial chaos expansion applied to eigenvalue problems.

The aim of this paper is to propose a non-intrusive approach to characterize a random eigenspace with a reduced number of deterministic finite element computations. A simple observation is at the origin of the proposed approach: in some cases, the eigenvalues of a structure with random parameters are random, but the corresponding eigenvectors are quite deterministic. In these cases, a simple deterministic computation is sufficient to characterize the random eigenmode. For the remaining eigenvalues, it is then necessary to use more accurate methods. This approach, referred to as the SMR method for Stochastic Model Reduction approach, consists in adapting stochastic modeling to each random eigenvalue depending on the global accuracy requirements on the whole set of random eigenvalues. Finally, the proposed approach minimizes the computational cost by concentrating computational resources on particular eigendata sets according to their configuration. The next section describes the SMR approach and its different refinement levels. As the proposed approach relies on the random eigenfrequencies configuration, an appropriate indicator referred to as the "proximity factor" is also developed in this section. Then a three-degrees-of-freedom test case illustrates the accuracy of the method, depending on the random eigenfrequencies configuration. We characterize the indicator for a large number of configurations. In the fourth section, the approach is applied to a more realistic system consisting of a frame with different random Young's moduli. Finally, an industrial structure is used to illustrate the efficiency of the SMR approach in terms of computation time and accuracy of statistical moments estimation.

2. Stochastic model reduction

2.1. Problem presentation

The general eigenvalue problem of undamped or proportionally damped systems can be expressed for a problem with n_d degrees of freedom by

$$\lambda_k(\theta)\mathbf{M}(\theta)\boldsymbol{\Phi}_k(\theta) = \mathbf{K}(\theta)\boldsymbol{\Phi}_k(\theta) \tag{1}$$

where

$$\lambda_k(\theta) \in \mathbb{R}, \quad \boldsymbol{\Phi}_k(\theta) \in \mathbb{R}^{n_d}, \quad \mathbf{M}(\theta) \in \mathbb{R}^{n_d \times n_d}, \quad \mathbf{K}(\theta) \in \mathbb{R}^{n_d \times n_d}, \quad \theta \in \Omega$$

λ_k and $\boldsymbol{\Phi}_k$ are the kth eigenvalue and the kth associated eigenvector. The relationship between the eigenvalues and the natural frequencies of the system is $\lambda_k = \omega_k^2$. The eigenvector $\boldsymbol{\Phi}_k$ is assumed to be mass normalized so that $\boldsymbol{\Phi}_k^\top \mathbf{M} \boldsymbol{\Phi}_k = 1$. $(\Omega, \mathcal{F}, p)$ is the abstract probability space associated with the underlying physical experiments. $\theta \in \Omega$ is a basic event from the complete probability space Ω. The space of square integrable random variables is denoted by $L_2(\Omega)$ and forms a Hilbert space with the norm $\|\cdot\|_{L_2(\Omega)}$. Matrices $\mathbf{M}(\theta)$ and $\mathbf{K}(\theta)$ represent the mass and stiffness matrices of the structure. Their randomness is due to the physical parameters of the structure such as mass density, Young's modulus or geometric

Please cite this article in press as: M. Ghienne et al., Stochastic model reduction for robust dynamical characterization of structures with random parameters, C. R. Mecanique (2017), https://doi.org/10.1016/j.crme.2017.09.006