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A folded plate clamped along one side only ☆

Une plaque pliée encastrée d'un côté seulement

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ABSTRACT

An asymptotic model of a folded thin elastic plate is posed on two plane domains and contains transmission conditions at the common line segment of their boundaries. These conditions become non-local and inhomogeneous if only one side of the plate is fixed. Solvability and smoothness results and error estimates for the model are derived.

RÉSUMÉ

On considère un modèle asymptotique de plaque mince élastique pliée reposant sur deux domaines plans et mettant en jeu des conditions de transmission à l'interface entre les deux domaines. Ces conditions deviennent non locales et inhomogènes lorsqu'un seul bord de la plaque est encastré. On fournit des résultats concernant le caractère bien posé du problème, on établit des résultats de régularité des solutions, et on prouve des estimations d'erreur pour le problème modèle.

1. Formulation of the problem

Let

$$\Omega_j^h = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : y^j \in \Sigma_j, |z| < h/2\}, j = 2, 3 \tag{1}$$

be two elastic plates with rectangular mid-sections

$$\Sigma_j = \{y^j = (y_1^j, y_2^j) : y_1^j \in (0, \ell_1), y_2^j \in (0, \ell_j)\}, \ell_j > 0, j = 2, 3 \tag{2}$$

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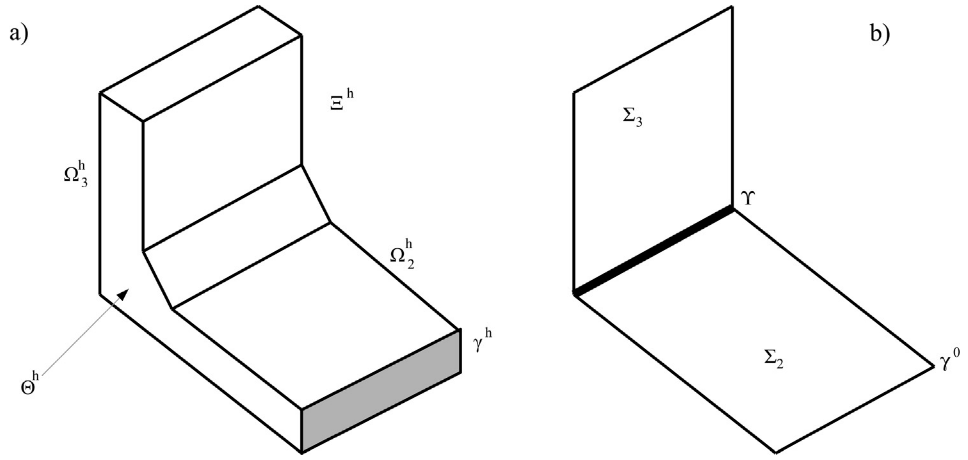

Fig. 1. The junction of plates (a) and its two-dimensional image (b).

where the local coordinates are given by $y^j = (x_1, x_j)$ and $z^2 = x_3, z^3 = x_2$. By rescaling, we set $\ell_1 = 1$ to make the coordinates and geometrical parameters dimensionless, in particular $h \ll 1$. The homogeneous isotropic elastic junction $\Xi^h = \Theta^h \cup \Omega_2^h \cup \Omega_3^h$ (Fig. 1a) consists of the plates (1) and the coupling bar

$$\Theta^h = \{x : x_1 \in (0, \ell_1), \xi = (\xi_2, \xi_3) = (h^{-1}x_2, h^{-1}x_3) \in \theta\} \tag{3}$$

where the domain $\theta \subset \mathbb{R}^2$ is bounded by a piecewise smooth contour $\partial\theta$ and includes the square $\{\xi : |\xi_j| < 1/2\}$ so that $\Omega_2^h \cap \Omega_3^h \subset \Theta^h$. The lateral side $\gamma^h = \{x : x_1 \in (0, \ell_1), x_2 = \ell_3, |x_3| < h/2\}$ (shaded in Fig. 1a) is clamped but the remaining part $\Gamma^h = \partial\Xi^h \setminus \gamma^h$ of the surface is traction-free. A volume force f^h is applied, e.g., due to gravity. The deformation of the solid Ξ^h is described by the boundary value problem

$$-\mu\Delta u^h - (\lambda + \mu)\nabla\nabla \cdot u^h = f^h \quad \text{in } \Xi^h \tag{4}$$

$$\sigma^{(n)}(u^h) = 0 \text{ on } \Gamma^h \setminus \Upsilon^h \tag{5}$$

$$u^h = 0 \text{ on } \gamma^h \tag{6}$$

where $\nabla =$grad, $\nabla\cdot =$div and Δ is the Laplace operator. Moreover, $\lambda \geq 0$ and $\mu > 0$ are the Lamé constants, $u = (u_1, u_2, u_3)$ is the displacement vector, and stresses are determined as follows:

$$\begin{aligned} &\sigma_{pq} = \mu(\partial_p u_q + \partial_q u_p) + \delta_{p,q}\lambda(\partial_1 u_1 + \partial_2 u_2 + \partial_3 u_3), \quad \partial_p = \partial/\partial x_p \\ &\sigma^{(n)} = (\sigma_1^{(n)}, \sigma_2^{(n)}, \sigma_3^{(n)}), \quad \sigma_p^{(n)} = n_1\sigma_{p1} + n_2\sigma_{p2} + n_3\sigma_{p3} \end{aligned} \tag{7}$$

Here, $\delta_{p,q}$ is the Kronecker symbol and $n = (n_1, n_2, n_3)$ is the unit vector of the outward normal defined everywhere on $\partial\Xi^h$, with the exception of the union Υ^h of closed edges on the surface.

The main goal of this paper is to present the asymptotics as $h \to +0$ of elastic fields in the folded plate Ξ^h clamped along the only lateral face γ^h.

2. Motivation

In the pioneering paper [1], the asymptotics of elastic fields in a cantilever, a junction of two perpendicular thin rods, is examined and the transmission conditions are derived at the joint point of two line segments substituting for the spacial rods in the model. The most intriguing observation in [1] is that the obtained one-dimensional models of the cantilevers with one or two clamped exterior ends differ from each other, namely they involve distinct groups of transmission conditions. Further effects of the interaction between elements of anisotropic thin elastic rod construction are found out in the paper [2], where the classification "fixed/movable" for rods and nodes in the junction is introduced and a procedure to detect its movable fragments is developed. The latter require a certain modification of asymptotic Ansätze for elastic fields and introduces new algebraic unknowns and orthogonality conditions for unknown functions on edges of the "skeleton" of the elastic junction. An adaptation of this procedure and asymptotic analysis of the spectral elasticity problem for various junctions of rods is given in [3,4].

Similar interaction effects can be readily predicted in thin-wall structures. This direction of mathematical studies is started with the paper [5], where a folded plate is considered, that is, a junction of two perpendicular thin straight plates Ω_2^h and Ω_3^h. The resultant transmission conditions at the common side Υ of the rectangles Σ_2 and Σ_3, i.e. the plate mid-sections, are derived under the assumption that both plates are fixed along the lateral sides, which are parallel to the