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An approximate strength criterion of porous materials with a pressure sensitive and tension-compression asymmetry matrix

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ABSTRACT

An approximate analytical criterion is established in the present study to describe the macroscopic strength behavior of porous materials with a pressure sensitive and tension-compression asymmetry solid matrix. The solid matrix is assumed to obey a Mises-Schleicher type criterion at the local scale. By using the stress variational homogenization method (SVH) with a strictly statically admissible stress field, an analytical criterion is first obtained. This criterion can retrieve the exact solution for the hydrostatic loading and provide a good prediction of the strength for the pure deviatoric loading (Σ_{eq}/σ_o). However, for small values of stress triaxiality in compressive domain, the analytical criterion underestimates the strength. Based on the accurate value of Σ_{eq}/σ_o obtained by the SVH method and taking into account some special conditions and requirements, an approximate improved strength criterion is derived. For a wide validation of the proposed criterion, new numerical results are presented based on the finite element simulations. The improved criterion is then verified through comparisons with both the upper and lower bounds given in Pastor, Kondo, and Pastor (2013) and the finite element results for a large range of porosity and tension-compression asymmetry ratio of the matrix. It is found that this new criterion improves existing ones. Finally, the criterion is applied to describe the plastic yield stresses of porous chalk and plaster.

1. Introduction

For many engineering materials such as rock-like materials, the tensile strength can be very different to the compressive one. For example, the uniaxial compression strength is much higher than the uniaxial tensile strength for concrete, rocks and ceramics. It is important to take into account this strength asymmetry in engineering design. A number of studies have been devoted to the description of deformation and strength of various materials with a tension-compression asymmetry (Barros, Alves, Oliveira, & Menezes, 2016; Cazacu, Plunkett, & Barlat, 2006; Cazacu & Stewart, 2009; Kim, Lee, Kim, & Lim, 2017; Shen, Kondo, Dormieux, & Shao, 2013; Shen, Shao, Kondo, & Gatmiri, 2012). On the other hand, most engineering materials contain voids or pores at different scales. The mechanical behavior of such materials is affected by the porosity. With the increase of porosity, the strength of porous materials significantly decreases. In the framework of limit analysis, a macroscopic strength criterion was established in Gurson (1977) for porous materials represented by hollow sphere or cylinder with a von Mises

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type incompressible solid matrix. The effect of porosity on the macroscopic strength is explicitly taken into account in this criterion. In order to improve the performance of the Gurson's model with respect to numerical results obtained from direct simulations, a number of heuristic modifications have been proposed by various authors, for instance Tvergaard and Needleman (1984) and Tvergaard (1981, 1990). Other extensions have also been developed for considering the influence of pore shape (Benzerga & Besson, 2001; Gologanu, Leblond, & Devaux, 1993; 1994; Gologanu, Leblond, Perrin, & Devaux, 1997; Keralavarma & Benzerga, 2010; Liao, 2004; Liao & Tang, 1997; Monchiet, Cazacu, Charkaluk, & Kondo, 2008) and the effect of matrix anisotropy (Monchiet et al., 2008; Monchiet, Charkaluk, & Kondo, 2014; Pardoen & Hutchinson, 2000). Moreover, in order to consider the pressure sensitivity of rock-like materials, some contributions have investigated the porous materials with a pressure-sensitive solid matrix, described for instance by a Drucker–Prager or Green type criterion (Durban, Cohen, & Hollander, 2010; Fritzen, Forest, Kondo, & Böhlke, 2013; Guo, Faleskog, & Shih, 2008; Jeong, 2002; Lee & Oung, 2000; Pastor, Kondo, & Pastor, 2013a; Shen, He, Dormieux, & Kondo, 2014; Shen, Pastor, & Kondo, 2013; Shen, Shao, Dormieux, & Kondo, 2012; 2012; Shen, Zhang, Shao, & Kondo, 2017; Trillat & Pastor, 2005).

However, some important scatters have been observed between theoretical predictions provided by previous criteria and numerical results issued from direct simulations and bounds given by limit analysis. On the other hand, the linear Drucker–Prager criterion is not able to correctly describe the strength asymmetry between tension and compression. In general, the tensile strength is largely overestimated. Therefore, further developments are needed. This work focuses on the formulation of a new strength criterion for a class of porous material exhibiting a large tension-compression strength asymmetry in the solid matrix. For this purpose and based on some previous studies (Aubertin & Li, 2004; Durban et al., 2010; Kovrizhnykh, 2004; Lee & Oung, 2000; Pastor, Kondo, & Pastor, 2013b; Raghava, Caddell, & Yeh, 1973; Zhang, Ramesh, & Chin, 2008), the parabolic Mises-Schleicher criterion (Schleicher, 1926) will be adopted to for the solid matrix. In this context, the exact solution of strength has been presented in Monchiet and Kondo (2012) for the particular case of pure hydrostatic loading. Numerical upper and lower bounds have also been reported in Pastor et al. (2013b) by means of 3D finite element calculations of the static and kinematic methods of limit analysis. The criteria respectively proposed by Lee and Oung (2000) and Durban et al. (2010) are not in good agreement with these results. Recently, the criterion derived by Shen, Shao, Kondo, and De Saxce (2015) improves those by Lee and Oung (2000) and Durban et al. (2010), but its prediction in the case of pure deviatoric loading should be improved.

In the present study, the stress variational homogenization method (SVH) is used to derive the macroscopic strength criterion of porous materials. At the first step, with a strict statically admissible stress field, a macroscopic criterion is formulated in Section 2. This criterion is assessed through comparisons with numerical results issued from direct finite element simulations. It is found that the criterion retrieves the exact solution for the hydrostatic loading, provides a good prediction for the pure deviatoric loading but gives an underestimated strength for the compression loading with a small stress triaxiality. At the second step, an improved approximate criterion is determined in Section 3 based on the analytical criterion issued from the SVH method. The improved criterion is assessed in Section 4 by the comparisons with both numerical upper and lower bounds (Pastor et al., 2013b) and finite element solutions obtained in this work. It is found that the new criterion provides a significant improvement of previous results. Finally, the proposed criterion is applied to describe the yield stresses of porous chalk and plaster.

2. Derivation of an analytical criterion

2.1. Problem statement

We shall consider here a porous material represented by a hollow sphere with a tension-compression asymmetry solid matrix. Its internal radius is a and the external one is b. Ω denotes the volume of the whole sphere, $|\Omega| = 4\pi b^3/3$, Ω_m is the volume of the solid matrix, whereas ω denotes the volume of the void, $|\omega| = 4\pi a^3/3$. The porosity of the hollow sphere is $f = a^3/b^3$. The solid matrix is isotropic, pressure sensitive and exhibits a tension-compression asymmetry. Its tensile yield stress is denoted by σ_T and the absolute compression yield stress is σ_C. To account for these properties, the Mises–Schleicher criterion (Lubliner, 1990; Schleicher, 1926) is adopted here to describe the local yield condition of the solid matrix :

$$\phi(\boldsymbol{\sigma}) = \sigma_{eq}^2 + 3\alpha\sigma_0\sigma_m - \sigma_0^2 \leq 0 \tag{1}$$

where $\sigma_m = \mathrm{tr}(\boldsymbol{\sigma})/3$ denotes the mean stress, $\sigma_{eq} = \sqrt{(3/2)\overline{\boldsymbol{\sigma}} : \overline{\boldsymbol{\sigma}}}$ is the equivalent shear stress with $\overline{\boldsymbol{\sigma}}$ being the deviatoric part of the stress tensor, $\overline{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \sigma_m \mathbf{I}$ and $\mathbf{I}$ the second-order unit tensor. α and σ_0 are two matrix property parameters. They are independent on the porosity f but both are related to the local tensile strength σ_T and the absolute compression strength σ_C ($\sigma_C \geq \sigma_T$) of the solid matrix by :

$$\alpha = \frac{\sigma_C - \sigma_T}{\sqrt{\sigma_C \sigma_T}}, \qquad \sigma_0 = \sqrt{\sigma_C \sigma_T} \tag{2}$$

The problem to be solved here is the determination of an analytical macroscopic strength criterion of the porous material represented by such a hollow cylinder.