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Buckling analysis of graphene-reinforced mechanical metamaterial beams with periodic webbing patterns

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ABSTRACT

Mechanical metamaterial beams (MMB) have been extensively studied given their potential functional applications in various areas, e.g. micro-electro-mechanical systems (MEMS), energy harvesting, and actuation. This study presents a novel class of graphene-reinforced MMB (GR-MMB) with arbitrarily periodic webbing. A size-dependent theoretical model is developed to predict and control the buckling response of the GR-MMB. The modified couple stress theory is expanded to include the effective material properties of microstructures. Clamped-clamped and simply supported GR-MMB with oval, hexagonal and cylindrical webbing patterns are showcased. Numerical simulations are conducted to validate the theoretical model and satisfactory agreements are obtained. Parametric studies are presented to unveil the effects of the graphene reinforcements and periodic design patterns on the buckling response of GR-MMB. The enhancement factor of the axial force between the GR-MMB and MMB, ψ, is studied with respect to the material ratio and geometric ratio. Density plots of the presented microstructures are provided to demonstrate the desired geometries that lead to the highest axial load and largest webbing diameter, i.e., lowest self-weight. The theoretical model presented in this study can be deployed to predict and tune the buckling response of GR-MMB with arbitrarily periodic webbing for different purposes.

1. Introduction

Sandwich structures are defined as two face sheets separated by a core to increase moment of inertia with little added weight, providing excellent resistance in bending and buckling loads. These structures have been used extensively in different applications, e.g., aircraft, portable structures, submarines, sports equipment, and nuclear reactors (Gibson & Ashby, 1999). The concerns regarding conventional sandwich structures can be summarized as: 1) how to improve critical buckling load while maintaining similar self-weight; and 2) how to prevent discontinuity stresses at the interface between face sheets and core, since they may lead to serious delamination issues (Sofiyev, 2017). To sufficiently address these issues, especially at the micro/nanoscale, current studies focus on three major directions:

- Materials strategies—Replacing traditional materials, for example metal or wood, with multifunctional materials such as carbon nanotube (CNT) composites, functionally graded materials (FGM), or porous materials (Sobhaniaragh, Ba-

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tra, Mansur, & Peters, 2017). Optimizing traditional materials using computational approaches to obtain tunable mechanical behaviors (Hu & Burgueno, 2018; Vogiatzis, Chen, Wang, Li, & Wang, 2017).
- Reinforcing/repairing strategies—Attaching reinforcement layers, such as carbon-based composites, to the face sheets of sandwich panels.
- Geometric strategies—Changing solid cores of traditional sandwich structures to cores with open, closed, or periodic cells (e.g., honeycombs with triangular, square, or hexagonal cells).

From a materials perspective, graphene-based reinforcements paired with a variety of matrix materials offer modulus enhancements of up to 100% due to a tensile in-plane elastic modulus of hundreds of GPa (Liu et al., 2016; Steurer, Wissert, Thomann, & Mülhaupt, 2009; Zhang et al., 2017). Such composite films have been developed for applications in mechanical, optical, electrical or biological fields (Yang, Hou, & Kotov, 2012). Graphene-based materials, e.g., orthotropic double-layered graphene sheets, have been theoretically investigated for enhancement in sandwich structures (Radic & Jeremic, 2017). From a reinforcing/repairing perspective, multifunctional microstructures specifically combine carbon-based materials, for example graphene-reinforced FGM (GR-FGM) (Mirzaei & Kiani, 2017; Zhao et al., 2017), or porous materials enhanced with graphene platelets (Sahmani, Aghdam, & Rabczuk, 2018). Theoretical studies have been carried out to predict the mechanical behaviors of those enhanced, sandwich-like structures in the materials perspective. Shear deformation of FGM sandwich conical shells subjected to axial loads was studied by Sofiyev (2017). Size-dependent Euler-Bernoulli beam models were reported to investigate the buckling response of FGM using the nonlocal strain gradient theory (Li & Hu, 2017; Li, Li, Hu, Ding, & Deng, 2017). A significant issue in solving the mechanical response of these graphene-enhanced sandwich-like structures, however, is to accurately obtain the effective material properties. One approach is to leverage the Halpin–Tsai micromechanical model to formulate the effective material properties of GR-FG laminated shells (Shen, Xiang, Lin, & Hu, 2017, 2018), and GR-FG porous micro/nanobeams (Sahmani et al., 2018). In these cases, the authors accounted for graphene platelets by introducing enhancement factors into the material properties. From a geometric perspective, macroscale beams with different shape configurations and constraints are investigated for multifunctional applications, namely energy harvesting or damage sensing (Borchani, Jiao, Burgueno, & Lajnef, 2017; Jiao, Borchani, Alavi, Hasni, & Lajnef, 2017a, b). Reducing to the micro/nanoscale, mechanical metamaterials with programmably periodic patterns have been attracting significant research interests for various applications, e.g., micro-electro-mechanical systems (MEMS), energy harvesting, or actuation (Zadpoor, 2016). Mechanical metamaterials exhibit controllable mechanical properties, e.g., Poisson's ratio or stress-strain curves, through designed and optimized periodic webbing patterns (Liu & Zhang, 2018; Zhang, Guo, Wu, Fang, & Zhang, 2018). Mechanical metamaterial beams (MMB), typically consisting of discontinuous face sheets and periodic webbings, are considered as a novel class of multifunctional microstructures as they can have high aspect and stiffness-to-weight ratios, and enhanced robustness over solid microbeams.

In this study, we investigate the mechanical characteristics of MMB reinforced by graphene layers. We report a novel class of graphene-reinforced MMB (GR-MMB) with arbitrarily periodic webbing. This study improves the advanced mechanical performance of MMB by combining the materials, geometric and reinforcing strategies. The novelty of this study could be summarized as: 1) reporting of a type of GR-MMB with arbitrarily periodic webbing designed at the micro/nanoscale; 2) proposing of an approach to consider the effective material properties of the GR-MMB; 3) developing of a size-dependent theoretical model to predict and control the buckling response of the GR-MMB; 4) presenting of a numerical model to validate the theoretical predictions; and 5) providing of parametric studies to demonstrate the effects of the graphene reinforcements and periodic patterns on the buckling response of the GR-MMB.

This study is outlined as: Section 2 proposes an approach to obtain the effective material properties of the GR-MMB with periodic webbing using the Halpin-Tsai method. Section 3 develops a size-dependent theoretical model using the modified couple stress theory. The effective material properties are substituted into the Euler-Bernoulli beam theory to gain the buckling response of the microstructures. In particular, GR-MMB with the oval, hexagonal and cylindrical webbing patterns are studied under the clamped-clamped (CC) and simply supported (SS) boundary conditions. Section 4 validates the theoretical results. The presented model is first simplified to microbeams, i.e., omitting the graphene reinforcements and periodic webbing, and compared with existing studies in the literature. FE simulations are then conducted to compare the force-displacement response and deflected configuration of the GR-MMB. Section 5 presents parametric studies to reveal the effects of the graphene enhancements and webbing patterns on the axial force. The maximum force difference between the MMB and GR-MMB is investigated with respect to the material ratio (i.e., Young's modulus of graphene-to-MMB $\frac{E^G}{E^{MMB}}$), and the geometric ratio (i.e., webbing diameter-to-beam width $\frac{D_2}{b}$). Density plots are provided to illustrate the optimal webbing diameter D_2 and Young's modulus of the graphene E^G to achieve the highest axial force with the lowest self-weight. Section 6 summarizes the main findings in this study. In the Appendices, we present the detailed derivations and results of the density plots.

2. Material properties of the GR-MMB with arbitrarily periodic webbing

Fig. 1 illustrates the novel class of GR-MMB with arbitrarily periodic webbing. The MMB consists of the length L, width b, height h, and thickness t^{MMB} (both the face sheets and webbing). The graphene layers are only attached to the face sheets of the MMB, which have the same the length and width as the MMB. The thickness of the graphene layers is denoted as t^G.