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An asymptotic strategy to couple homogenized elastic structures

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ABSTRACT

A two-scale methodology to calculate the local stress-strain state (SSS) in structures composed of connected elements is proposed. The methodology is based on the assumption that the connecting unit has a size small in comparison to the objects being connected. It is demonstrated that the problem of connection allows asymptotic decomposition. At the macroscopic level (the zero order approximation), an interface problem, with appropriate interface conditions, is revealed. At this order, the individual properties of the joint are neglected. These properties manifest themselves at the next asymptotic order, which takes into account all individual joint properties using the solution of the macroscopic problem. The local SSS in the vicinity of joint consists of the SSS in the connecting unit, together with rapidly decaying boundary layers in the connected elements. A detail elucidation of the local SSS in the connecting unit is an important distinction of this work from previous studies of connected structures. Motivated by the asymptotic analysis, a numerical method for simultaneously calculating the SSS in both the connected structures and the connecting unit is developed. An illustrative example, involving computation of the SSS in the vicinity of an explosion welding seam, is presented.

1. Introduction. Models of joint – interface surface or transition zone

Structural components composed of connected elements are common place in engineering practice. Two basic approaches, both based on models involving an interface surface and connecting unit or transition zone, are typically employed to determine mechanical response. In the first, the two elements of the structure are assumed to be directly adjacent to some interface surface, described by an "interface condition" on the surface. Often ideal contact conditions are used, which in the elastic case take the form

$$[\mathbf{u}] = 0, [\boldsymbol{\sigma}_n] = 0, \tag{1}$$

where [] denotes a jump on the interface surface, $\mathbf{u}$ is the displacement vector and $\boldsymbol{\sigma}_n$ is the stress vector normal to the interface surface. These conditions are often formulated on a weld seam and have also been used for riveted and glued joints

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(Bickford & Nassar, 1998; Cherepanov & Rybakov, 1977; Composite Materials and Joining Technologies for Composites 2012; Joining Technologies for Composites and Dissimilar Materials 2016). This approach can be oversimplified: for example, as conditions (1) do not contain parameters of the actual weld, a weld seam with a complex physical and geometric structure can only be treated as a surface without any individual properties. However, this idealization, by virtue of its simplicity, has been widely used and still proved to be useful for calculating the stress-strain state (SSS) in connected structural elements and within the vicinity of the joint. A second approach involves a detailed study of the interconnecting unit (e.g. the bolts in a bolted connection) or transition zone (e.g. welded seam) within the framework of the spatial elasticity theory problem (Neuss-Radu, 2000; Pietraszkiewicz & Konopicska, 2015; Rodabaugh & Atterbury, 1962). This approach allows one to take into account the physical and geometric characteristics of the connection and calculate the SSS in all structural elements. In this case, the connecting unit is often regarded as an isolated and independent element, not associated with the entire structure.

When trying to treat a connecting unit as part of the entire structure, we arrive at the following problem. Denote by $u_0^-(\mathbf{x})$ and $u_0^+(\mathbf{x})$, displacements in the main parts of the connected structural elements, i.e. in the parts of connected structural elements placed far from the joint. These solutions usually have simple form.

The solutions $u_0^-(\mathbf{x})$ and $u_0^+(\mathbf{x})$ usually do not satisfy conditions (1). This discrepancy is compensated for by the introduction of boundary layers (BLs) $\zeta^-(\mathbf{x})$ and $\zeta^+(\mathbf{x})$, which exhibit rapid decay away from the interface surface in domains P^- and P^+, respectively. This corresponds to an idealized contact scheme suitable for some practical situations, for example, when two elements are connected by compression (diffusion welding). The mathematical technique for coordinating the functions $u_0^-(\mathbf{x})+\zeta^-(\mathbf{x})$ and $u_0^+(\mathbf{x})+\zeta^+(\mathbf{x})$, and for the satisfaction of the contact condition, is well developed (see, e.g. Bauer, Filippov, Smirnov, Tovstik, & Vaillancourt, 2015, and references herein). In respect of the interface problem for composites, several authors have addressed this problem (see for example Allaire & Amar, 1999; Bensoussan, Lions, & Papanicolau, 1979; Dumontet, 1986; Neuss-Radu, 2000; Panasenko, 1981; Sanchez-Palencia, 1987 among others).

In the connecting unit, or transition zone model, there is a region P_0 between the elements P^- and P^+, within which the solution of the problem has the form:

$$\begin{aligned} &u(\mathbf{x}) = u_0^-(\mathbf{x}) + \zeta^-(\mathbf{x}) \text{ at } P^-;\\ &u(\mathbf{x}) = \varphi(\mathbf{x}) \text{ at } P_0;\\ &u(\mathbf{x}) = u_0^+(\mathbf{x}) + \zeta^+(\mathbf{x}) \text{ at } P^+. \end{aligned} \tag{2}$$

The elasticity theory problem for the functions (2) can be solved separately in each domain, P_0, P^- and P^+,with subsequent matching of the solutions. However, a simpler way is to employ the boundary perturbation method and the method of local perturbations (Panasenko, 1981, 2005; Pietraszkiewicz & Konopicska, 2015). We briefly describe the main idea of this approach.

In many cases, the sizes of connecting units, or transition zones, are small in comparison to the size of the connected structural elements. Let us denote the relative size of the connecting units by ε. The local perturbation theory is based on representation of the solution of the elasticity problem in the form

$$\mathbf{u_0}(\mathbf{x}) + \varepsilon \mathbf{u_1}(\mathbf{x}, \mathbf{x}/\varepsilon). \tag{3}$$

In (3), $\mathbf{u}^0(\mathbf{x})$ is the smooth leading order solution and $\varepsilon\mathbf{v}(\mathbf{x}, \mathbf{x}/\varepsilon)$ its corrector. In our approach, the corrector $\mathbf{v}(\mathbf{x}, \mathbf{y})$ takes the form of a local perturbation. We call a function "the local perturbation, corresponding to a perturbation of the geometry of the domain or material property of material occupying the domain", if it tends to zero as the distance between the actual point and the aforementioned domain tends to infinity.

Note that Ansatz (3) is well known from homogenization theory (Bakhvalov & Panasenko, 1989; Kalamkarov & Kolpakov, 1997; Sanchez-Palencia, 1980). However, in homogenization theory, the corrector is periodic in $\mathbf{y}$. This difference leads to drastic differences in the limit models.

The local perturbation method, on the one hand, explains the difference between the interface model and the connective unit model (transition zone), and on the other hand, demonstrates the consistency of the models. The interface model describes the global behavior of the solution, the model of connecting unit (transition zone) also describes the local perturbation of the solution. The interface model solution, obtained using the relative size of the transition zone as a small parameter, is the first term of the asymptotic solution of the original problem. This explains the effectiveness of the interface model for the analysis of the macroscopic behavior of connected bodies despite the neglecting a detailed analysis of the interface. A similar, but not identical problem arises in the multi-element thin-walled structures (see, e.g. Kozlov, Maz'ya, & Movchan, 1999; Panasenko, 2005; Zhikov, 2002; Zhikov & Pastukhova, 2003), where the structural mechanics model occurs within the zero-term of the asymptotic solution to the elasticity theory problem.

The paper is organized as follows. In Section 2, we develop an asymptotic analysis for homogeneous joined bodies. Section 3 illustrates an application of the local perturbation method to the calculation of the local SSS in a welded seam arising as a result of explosion welding of homogeneous materials. In Section 4, we expand the local perturbation method to the case of inhomogeneous (composite) joined bodies. Section 5 illustrates the application of the local perturbation method to the calculation of local SSS in a welded seam arising as a result of explosion welding of homogeneous and composite materials. In Section 6, we determine the minimal fragment of the connected elements adjacent to the connecting unit needed so that the solution in the selected zone approximates the solution of the original problem with sufficient accuracy. Lattice-solid connection is studied in Section 7. Finally, some concluding remarks are presented in Section 8.